

# NDARC — NASA Design and Analysis of Rotorcraft Theoretical Basis and Architecture

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## ABSTRACT

The theoretical basis and architecture of the conceptual design tool NDARC (NASA Design and Analysis of Rotorcraft) are described. The principal tasks of NDARC are to design (or size) a rotorcraft to satisfy specified design conditions and missions, and then analyze the performance of the aircraft for a set of off-design missions and point operating conditions. The aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. For each component, attributes such as performance, drag, and weight can be calculated. The aircraft attributes are obtained from the sum of the component attributes. NDARC provides a capability to model general rotorcraft configurations, and estimate the performance and attributes of advanced rotor concepts. The software has been implemented with low-fidelity models, typical of the conceptual design environment. Incorporation of higher-fidelity models will be possible, as the architecture of the code accommodates configuration flexibility, a hierarchy of models, and ultimately multidisciplinary design, analysis and optimization.

## INTRODUCTION

The objectives of rotorcraft design work in a government laboratory are to support research and to support rotorcraft acquisition. Research activities require a robust design capability to aid in technology impact assessments and to provide system level context for research. At the applied research level, it is necessary to show how technology will impact future systems, and justify the levels of investment required to mature that technology to an engineering development stage. Design provides one avenue to accomplishing these objectives. The Department of Defense (DoD) acquisition phases requiring rotorcraft design work include concept exploration, concept decision, concept refinement, and technology development. During these acquisition phases, performing quantitative evaluation and independent synthesis of a wide array of aircraft designs is typically necessary, in

order to provide the foundation for specification and requirement development.

Rotorcraft conceptual design consists of analysis, synthesis, and optimization to find the best aircraft meeting the required capabilities and performance. A conceptual design tool is used for synthesis and analysis of rotorcraft. These tools historically have been low fidelity for rapid application. Such sizing codes are built around the use of momentum theory for rotors, classical finite wing theory, a referred parameter engine model, and semi-empirical weight estimation techniques. The successful use of a low-fidelity tool requires careful consideration of model input parameters and judicious comparison with existing aircraft to avoid unjustified extrapolation of results.

The helicopter industry has proprietary conceptual design tools, including PRESTO (Bell Helicopter), RDM (Sikorsky Aircraft), and HESCOMP and VASCOMP (Boeing). Until now the tools available to the U.S. government have been characterized by out-of-date software and limited capabilities. Examples are HESCOMP and VASCOMP (the versions developed by

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Boeing in the 1970s), and RC (developed by the U.S. Army AFDD in the 1990s).

NASA, with support from the U.S. Army, conducted in 2005 the NASA Heavy Lift Rotorcraft Systems Investigation (ref. 1), focused on the design and in-depth analysis of rotorcraft configurations that could satisfy the Vehicle Systems Program (VSP) technology goals. The VSP technology goals and mission were intended to identify enabling technology for civil application of heavy lift rotorcraft. The goals emphasized efficient cruise and hover, efficient structure, and low noise. The requirements included carrying 120 passengers over a 1200 nm range, 350 knots at 30,000 ft altitude. The configurations considered included the Large Civil Tiltrotor (LCTR), Large Civil Tandem Compound (LCTC), and Large Advancing Blade Concept (LABC). This project is an example of the role of a rotorcraft sizing code within a government laboratory. The design tool used was the AFDD RC code. The project illustrated the difficulties adapting or modifying a legacy code for configurations other than conventional helicopters and tiltrotors.

Since 2005, there have been numerous other joint NASA/U.S. Army investigations of advanced rotorcraft concepts, covering conventional tiltrotors and helicopters, slowed-rotor compound helicopters (ref. 2), a tilting-tandem concept, heavy-lift slowed-rotor tiltrotors (ref. 3), lift-offset rotor concepts (ref. 4), and a second generation large civil tiltrotor (LCTR2, ref. 5). These design projects have gone well beyond the conventional boundaries of the conceptual design process, combining high-fidelity analyses (including rotorcraft comprehensive analysis, computational fluid dynamics, and structural analysis) with the conceptual design tool. This approach has been required because of the increasing sophistication of the requirements and the technology, and the increased level of certainty needed to differentiate between system concepts.

Based on this experience, a new conceptual design tool has been developed to support future needs of the NASA Subsonic Rotary Wing project and the U.S. Army AFDD Advanced Design Office: NASA Design and Analysis of Rotorcraft (NDARC). The software development started in January 2007, and the initial code release occurred in May 2009. This paper summarizes the NDARC theoretical basis and architecture; the complete description is in reference 6. A companion paper (ref. 7) presents validation and demonstration results from the NDARC development.

## REQUIREMENTS

Based on the recent experience of NASA and AFDD at Ames Research Center, the following requirements were defined for the new conceptual design tool.

The principal tasks of the tool are to design rotorcraft to meet specified requirements, and then analyze the performance of the aircraft for a set of flight conditions and missions. Multiple and flexible design requirements, from specific flight conditions and various missions, must be accommodated in the sizing task. The aircraft performance analysis must cover the entire spectrum of the aircraft capabilities, and component performance and engine models must cover all operating conditions. A general and flexible capability to define conditions and missions is required.

For government applications and to support research, it is important to have the capability to rapidly model general rotorcraft configurations, including estimates of the performance and attributes of advanced rotor concepts and the capability to model the impact of new technology at the system and component level. In such an environment, software extensions and modifications will be routinely required to meet the unique requirements of individual projects. Complete and thorough documentation of the theory and its software implementation is essential, to support development and maintenance and to enable effective use and modification.

The code architecture must accommodate configuration flexibility and alternate models, including a hierarchy of model fidelity. Although initially implemented with low-fidelity models, typical of the conceptual design environment, ultimately the architecture must allow multidisciplinary design, analysis, and optimization.

Definition and development of the NDARC requirements benefited substantially from the experiences and computer codes of the preliminary design team of the U.S. Army Aeroflightdynamics Directorate (AFDD) at Ames Research Center. This background is described in reference 6. In the early 1990s, the RC code (for RotorCraft) emerged from the AFDD efforts, with RC97 a major version that unified tiltrotor and helicopter analyses. NDARC is entirely new software, built on a new architecture for the design and analysis of rotorcraft. From the RC theoretical basis, the equations of the parametric weight equations and the Referred Parameter Turbo shaft Engine Model were used with only minor changes. Use was also made of some RC component aerodynamic models and rotor performance models. The current users

of RC, informed by past and recent applications, contributed significantly to the requirements definition.

### NDARC TASKS

The NDARC code performs design and analysis tasks. The design task sizes the rotorcraft to satisfy specified design conditions and missions. The analysis tasks can include off-design mission performance analysis, flight performance calculation for point operating conditions, and generation of subsystem or component performance maps. Figure 1 illustrates the tasks. The principal tasks (sizing, mission analysis, flight performance analysis) are shown in the figure as boxes with heavy borders. Black arrows show control of subordinate tasks.

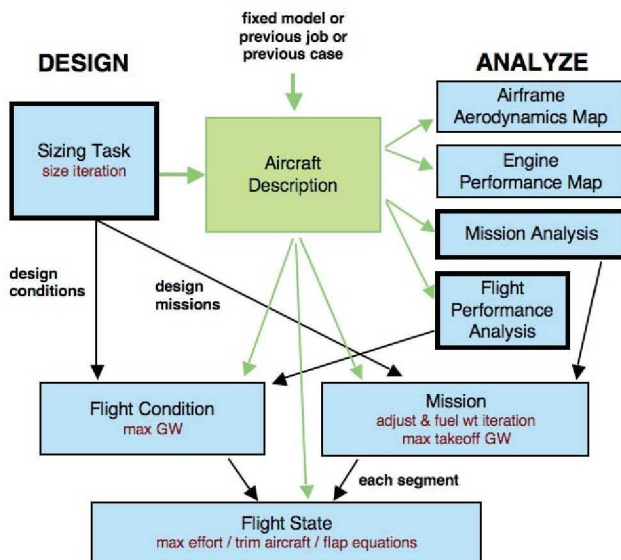


Figure 1. Outline of NDARC tasks.

The aircraft description (figure 1) consists of all the information, input and derived, that defines the aircraft. This information can be the result of the sizing task; can come entirely from input, for a fixed model; or can come from the sizing task in a previous case or previous job. The aircraft description information is available to all tasks and all solutions (indicated by green arrows).

The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a specified set of design conditions and missions. The aircraft size is characterized by parameters such as design gross weight, weight empty, rotor radius, and engine power available. The relationships between dimensions, power, and weight

generally require an iterative solution. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized.

Missions are defined for the sizing task and for the mission performance analysis. A mission consists of a specified number of mission segments, for which time, distance, and fuel burn are evaluated. For the sizing task, certain missions are designated for design gross weight calculations, for transmission sizing, and for fuel tank sizing. The engines are sized to meet the power requirement of each segment. The mission parameters include mission takeoff gross weight and useful load. For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is on time or distance (if adjustable), or on fuel weight.

Flight conditions are specified for the sizing task and for the flight performance analysis. For the sizing task, certain flight conditions are designated for design gross weight calculations, for transmission sizing, for maximum takeoff weight calculations, and for antitorque or auxiliary-thrust rotor sizing. The engines are sized to meet the power requirement of each condition. The flight condition parameters include gross weight and useful load.

For flight conditions and mission takeoff, the gross weight can be maximized such that the power required equals the power available.

A flight state is defined for each mission segment and each flight condition. The aircraft performance can be analyzed for the specified state, or a maximum-effort performance can be identified. The maximum effort is specified by identifying a target (such as best endurance, best range, or power required equal power available) to be achieved by adjusting a variable (such as speed, rate of climb, or altitude).

The aircraft must be trimmed by solving for the controls and motion that produce equilibrium in the specified flight state. Different trim solution definitions are required for various flight states, hence for each mission segment and flight condition.

Evaluating the rotor hub forces and blade pitch angles may require solution of the blade flap equations of motion.

The following sections describe the NDARC tasks in more detail.

### AIRCRAFT DESCRIPTION

Decomposition of the aircraft system into fundamental components is critical to achieving the ability to rapidly model a wide array of rotorcraft concepts. Thus the aircraft consists of a set of components, including fuselage, rotors, wings, tails, and propulsion. For each component, attributes such as performance, drag, and weight can be calculated. The aircraft attributes are obtained from the sum of the component attributes. Description and analysis of conventional rotorcraft configurations is facilitated, while retaining the capability to model novel and advanced concepts. Specific rotorcraft configurations considered are single main-rotor and tail-rotor helicopter; tandem helicopter; coaxial helicopter; and tiltrotors. Novel and advanced concepts typically are modeled by starting with one of these conventional configurations. For example, compound rotorcraft can be constructed by adding wings and propellers.

The following components form the aircraft.

- a) Systems: The systems component contains weight information (fixed useful load, vibration, contingency, systems and equipment) for the aircraft.
- b) Fuselage: There is one fuselage for the aircraft.
- c) Landing Gear: There is one landing gear for the aircraft.
- d) Rotors: The aircraft can have one or more rotors, or no rotors. In addition to main rotors, the component can model tail rotors, propellers, proprotors, and ducted fans. A rotor is designated a main rotor, tail rotor, or propeller; and can be tilting, ducted, and/or antitorque. The rotor power required is evaluated using the energy method, as a sum of induced, profile, and parasite power. The power components are calculated in terms of an induced power factor and a mean drag coefficient. The power models account for the influence of speed, thrust, compressibility, stall, lift offset, and the induced interference between twin rotors. The calibration of these induced and profile power models reflects the level of technology being considered. Blade element theory is used to calculate rotor hub forces and moments and to solve for blade pitch control or flapping.
- e) Forces: The force component is a simple model for a lift, propulsion, or control subsystem, including a weight and fuel flow description.

f) Wings: The aircraft can have one or more wings, or no wings.

g) Tails: The aircraft can have one or more horizontal or vertical tail surfaces, or no tails.

h) Fuel Tank: There is one fuel tank component for the aircraft. There are one or more sizes of auxiliary fuel tanks.

i) Propulsion Groups: There are one or more propulsion groups. Each propulsion group is a set of rotors and engine groups, connected by a drive system. The components define the power required, and the engine groups define the power available. There are one or more drive states, with a set of gear ratios for each state. The power required equals the sum of component power, transmission losses, and accessory losses. There are drive system torque limits, and rotor and engine shaft ratings.

j) Engine Groups: Each engine group has one or more engines of the same type. For each engine type an engine model is defined.

The engine model describes a particular engine, used in one or more engine groups. The engine performance information includes mass flow, fuel flow, jet thrust, and momentum drag at the required power. A Referred Parameter Turbohaft Engine Model (RPTM) enables the aircraft performance analysis to cover the entire spectrum of operation. This model uses curve fits of referred performance from an engine deck, including the effect of turbine speed. The effects of size (a scaling model, based on mass flow) and technology (specific power and specific fuel consumption) are included in the engine model.

The ability to define the aircraft control structure through input is a key feature for configuration generality. Aircraft controls are connected to component controls. Aircraft controls consist of pilot's controls (for trim), configuration variables (e.g. tilt of nacelle/pylon, engine, rotor shaft), and direct connections to component controls. There can be one or more control states, with different connections to components (for example, to model the controls of a tiltrotor in helicopter mode and airplane mode flight). There are default control connections for each configuration.

Weights are calculated or input for all components and subsystems. Calculated weights are obtained from parametric equations based on weights of existing turbine powered helicopters and tiltrotors (and some fixed wing aircraft component weights). Multiplicative technology factors can be used for the weights of all elements of the

aircraft, to either match measured weights or account for advanced technology.

### SIZING TASK

The sizing task determines the dimensions, power, and weight of a rotorcraft that can perform a specified set of design conditions and missions. The aircraft size is characterized by parameters such as design gross weight  $W_D$  or weight empty  $W_E$ , rotor radius  $R$ , and engine power available  $P_{eng}$ . The relationships between dimensions, power, and weight generally require an iterative solution. From the design flight conditions and missions, the task can determine the total engine power or the rotor radius (or both power and radius can be fixed), as well as the design gross weight, maximum takeoff weight, drive system torque limit, and fuel tank capacity. For each propulsion group, the engine power or the rotor radius can be sized. Alternatively,  $P_{eng}$  and  $R$  can be input rather than sized. Several aircraft parameters can be determined by a subset of the design conditions and missions:

- a) Design gross weight  $W_D$ .
- b) Maximum takeoff gross weight  $W_{MTO}$ .
- c) Drive system torque limit  $P_{DS\ limit}$ .
- d) Fuel tank capacity  $W_{fuel-cap}$ .
- e) Antitorque or auxiliary-thrust rotor design thrust  $T_{design}$ .

Alternatively, these parameters can be fixed at input values. The design gross weight  $W_D$  can be fixed. The weight empty can be fixed (achieved by adjusting the contingency weight).

For each flight condition and for each mission, the gross weight and useful load are specified. The gross weight can be input, or maximized, or fallout. For flight conditions, the payload or fuel weight can be specified, and the other calculated; or both payload and fuel weight specified, with gross weight fallout. For missions, the payload or fuel weight can be specified, the other fallout, and then time or distance of mission segments adjusted; or fuel weight calculated from mission, and payload fallout; or both payload and fuel weight specified (or payload specified and fuel weight calculated from mission), with gross weight fallout.

### Component Sizing

Numerous choices are implemented for identification of independent (fixed) and dependent (fallout) design parameters, with parameter variation facilitated by automating dependencies.

Engine Power. The engine size is described by the power  $P_{eng}$ , which is the sea-level static power available per engine at a specified takeoff rating.

Main Rotor. The main rotor size is defined by the radius  $R$  or disk loading  $W/A$ , thrust-weighted solidity  $\sigma$ , hover tip speed  $V_{tip}$ , and blade loading  $C_W/\sigma = W / \rho A V_{tip}^2$ . With more than one main rotor, the disk loading and blade loading are obtained from an input fraction of design gross weight,  $W = f_W W_D$ .

If the rotor radius is fixed for the sizing task, three of  $R$  (or  $W/A$ ),  $C_W/\sigma$ ,  $V_{tip}$ , and  $\sigma$  are input; and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor.

If the sizing task determines the rotor radius ( $R$  and  $W/A$ ), then two of  $C_W/\sigma$ ,  $V_{tip}$ , and  $\sigma$  are input; and the other parameter is derived. The radius can be sized for just a subset of the rotors, with fixed radius for the others.

Antitorque or Auxiliary-thrust Rotor. For antitorque and auxiliary-thrust rotors, three of  $R$  (or  $W/A$ ),  $C_W/\sigma$ ,  $V_{tip}$ , and  $\sigma$  are input; and the other parameters are derived. Optionally the radius can be calculated from a specified ratio to the radius of another rotor. The disk loading and blade loading are based on  $T_{design}$ , where  $T_{design}$  is the maximum thrust from designated design conditions. Optionally the tail rotor radius can be scaled with the main rotor radius and the disk loading.

Wing. The wing size is defined by the wing area  $S$  or wing loading  $W/S$ ; span (perhaps calculated from other geometry), chord, and aspect ratio. With more than one wing, the wing loading is obtained from an input fraction of design gross weight,  $W = f_W W_D$ .

Two of the following parameters are input: area (or wing loading), span, chord, and aspect ratio; the other parameters are derived. The span can be calculated from the rotor radius, fuselage width, and clearance (typically used for tiltrotors). The span can be calculated from a specified ratio to the span of another wing.

Tail. The tail size is determined by the area or tail volume, span, chord, and aspect ratio.

Fuel Tank. The fuel tank capacity  $W_{fuel-cap}$  (maximum usable fuel weight) is determined from designated sizing missions.

Weights. The structural design gross weight  $W_{SD}$  and maximum takeoff weight  $W_{MTO}$  can be input, or specified as  $d + fW$ , for input increment  $d$  and fraction  $f$ . This convention allows the weights to be input directly ( $f = 0$ ),

or scaled with  $W_D$ . For  $W_{SD}$ ,  $W$  is the design gross weight  $W_D$ , or  $W_D$  adjusted for a specified fuel state (input fraction of fuel capacity). For  $W_{MTO}$ ,  $W$  is the design gross weight  $W_D$ , or  $W_D$  adjusted for maximum fuel capacity. Alternatively,  $W_{MTO}$  can be calculated as the maximum gross weight possible at a designated sizing flight condition.

**Drive System Rating.** The drive system rating is defined as a power limit,  $P_{DS\text{limit}}$ . The rating is properly a torque limit,  $Q_{DS\text{limit}} = P_{DS\text{limit}} / \Omega$ , but is expressed as a power limit for clarity. The drive system rating can be input; calculated from the engine takeoff power rating; calculated from the power available at the transmission sizing flight conditions; or calculated from the power required at the transmission sizing flight conditions. The drive system rating is a limit on the entire propulsion system. To account for differences in the distribution of power through the drive system, limits are also used for the torque of each rotor shaft and of each engine group.

## MISSION DEFINITION

Missions are defined for the sizing task and for the mission performance analysis. A mission consists of a specified number of mission segments. The takeoff gross weight is evaluated at the start of the mission, perhaps maximized for zero power margin at a specified mission segment. Then the aircraft is flown for all segments. For calculated mission fuel weight, the fuel weight at takeoff is adjusted to equal the fuel required for the mission (burned plus reserve). For specified takeoff fuel weight with adjustable segments, the mission time or distance is adjusted so the fuel required for the mission (burned plus reserve) equals the takeoff fuel weight. The mission iteration is thus on mission fuel weight. A successive substitution method is used if an iteration is required.

Each mission consists of a specified number of mission segments. The following segment types can be specified.

- a) Taxi or warm-up (fuel burned but no distance added to range).
- b) Distance: fly segment for specified distance (calculate time).
- c) Time: fly segment for specified time (calculate distance).
- d) Hold: fly segment for specified time (loiter, so fuel burned but no distance added to range).
- e) Climb: climb or descend from present altitude to next segment altitude (calculate time and distance).
- f) Spiral: climb or descend from present altitude to next

segment altitude, fuel burned but no distance added to range.

The number of auxiliary fuel tanks can change with each mission segment. The aircraft can refuel (either on the ground or in the air) at the start of a mission segment, by either filling all tanks to capacity or adding a specified fuel weight. Fuel can be dropped at the start of a mission segment. For calculation of the time or distance in a mission segment, a headwind or tailwind can be specified.

Mission fuel reserves can be specified in several ways for each mission. Fuel reserves can be defined in terms of specific mission segments. Fuel reserves can be an input fraction of the fuel burned by all (except reserve) mission segments, or an input fraction of the fuel capacity.

The takeoff distance can be calculated, either as ground run plus climb to clear an obstacle, or accelerate-stop distance in case of engine failure. Landing and VTOL takeoff calculations are not implemented, as these are best solved as an optimal control problem.

## FLIGHT STATE

A flight state is defined for each flight condition and for each mission segment. The flight state definition consists of the speed, aircraft motion, altitude, atmosphere, height above ground level and landing gear state, aircraft control state, aircraft control values, and center-of-gravity position. Parameters defined for each propulsion group include drive system state, and rotor tip speed for primary rotor. Specified for each engine group are the number of inoperative engines, the infrared suppressor state, the engine rating, and the fraction of rated engine power available. Aircraft and rotor performance parameters for each flight state include payload drag, rotor performance, and aircraft trim state and trim targets.

The aircraft performance can be analyzed for the specified state, or a maximum-effort performance can be identified. The maximum effort is specified by identifying a target (such as best endurance, best range, or power required equal power available) to be achieved by adjusting a variable (such as speed, rate of climb, or altitude). Two maximum effort quantity/variable pairs can be specified, solved in nested iterations.

## Environment and Atmosphere

The aerodynamic environment is defined by the speed of sound  $c_s$ , and density  $\rho$ , and kinematic viscosity  $\nu = \mu / \rho$  of the air (or other fluid). These quantities can be obtained from the standard day (International Standard Atmosphere), or input directly. The International Standard



Atmosphere (ISA) is a model for the variation with altitude of pressure, temperature, density, and viscosity, published as International Standard ISO 2533 by the International Organization for Standardization (ISO) (ref. 8). The ISA consists of a series of altitude ranges with constant lapse rate (linear temperature change with altitude).

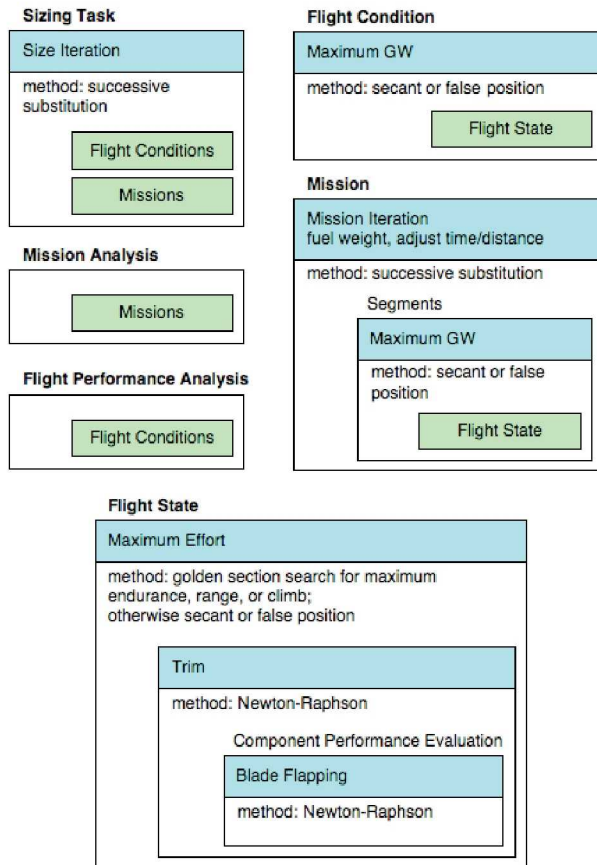


Figure 2. Design and analysis tasks, with nested loops and solution methods.

## SOLUTION PROCEDURE

The NDARC code performs design and analysis tasks. Figure 1 illustrates the tasks. The principal tasks (sizing, mission analysis, flight performance analysis) are shown in the figure as boxes with dark borders. Black arrows show control of subordinate tasks. The aircraft description consists of all the information, input and derived, that defines the aircraft. This information can be the result of the sizing task; can come entirely from input, for a fixed model; or can come from the sizing task in a previous case or previous job. The aircraft description information is

available to all tasks and all solutions (indicated by green arrows in figure 1).

The nested iteration loops involved in the solution process are indicated by the subtitles in the boxes of figure 1, and illustrated in more detail in figure 2. The flight state solution involves up to three loops. The innermost loop is the solution of the blade flap equations of motion, needed for an accurate evaluation of the rotor hub forces. The next loop is the trim solution, which is required for most flight states. The flight state optionally has one or two maximum effort iterations. The flight state solution is executed for each flight condition and for each mission segment. A flight condition solution or any mission segment solution can optionally maximize the aircraft gross weight. The mission usually requires an iterative solution, for fuel weight or for adjustable segment time or distance. Thus each flight condition solution involves up to four nested iterations: maximum gross weight (outer), maximum effort, trim, and blade motion (inner). Each mission solution involves up to five nested iterations: mission (outer), then for each segment maximum gross weight, maximum effort, trim, and blade motion (inner). Finally, the design task introduces a sizing iteration, which is the outermost loop of the process. The solution method for each iteration is indicated in figure 2. Details of the solution methods and their implementation are given in reference 6.

## COST

Costs are estimated using statistical models based on historical aircraft price and maintenance cost data, with appropriate factors to account for technology impact and inflation. The aircraft flyaway cost consists of airframe, mission equipment package, and flight control electronics costs. The direct operating cost plus interest (DOC+I, in cents per available seat mile) is the sum of maintenance cost, flight crew salary and expenses, fuel and oil cost, depreciation, insurance cost, and finance cost. Inflation factors can be input, or internal factors used (either DoD deflators or CPI factors).

The CTM rotorcraft cost model (refs. 9 to 11) gives an estimate of aircraft flyaway cost and direct operating cost plus interest. The statistical equation for airframe cost predicts the price of 123 out of 128 rotorcraft within 20% (figure 3). The fuel burn, block time, and block range are obtained from a designated mission.

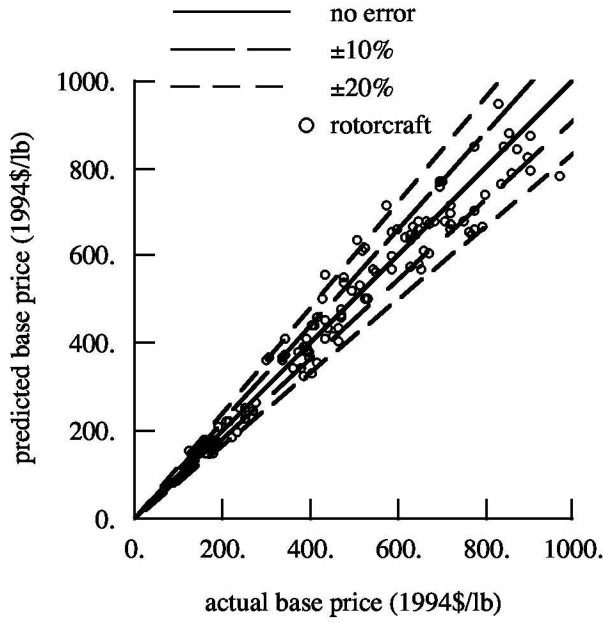


Figure 3. Statistical estimation of rotorcraft flyaway cost.

### AIRCRAFT MODEL

The aircraft disk loading is the ratio of the design gross weight and a reference rotor area:  $DL = W_D / A_{ref}$ . The reference area is a sum of specified fractions of the rotor areas,  $A_{ref} = \sum f_A A$ . Typically  $A_{ref}$  is the projected area of the lifting rotors, where the lifting rotors are all those not designated antitorque or auxiliary-thrust. The aircraft wing loading is the ratio of the design gross weight and a reference wing area:  $WL = W_D / S_{ref}$ . The reference area is a sum of the wing areas,  $S_{ref} = \sum S$ .

### Aircraft Controls

A set of aircraft controls  $c_{AC}$  are defined, and connected to the component controls. The connection to the component control  $c$  is typically of the form  $c = STc_{AC} + c_0$ , where  $T$  is an input matrix and  $c_0$  the component control for zero aircraft control. The factor  $S$  is available for internal scaling of the matrix. Figure 4 illustrates the control relationships.

The connection (matrix  $T$ ) is defined for several control system states, allowing change of control configuration with flight state. An example is control of a tiltrotor in helicopter mode and airplane mode flight. The control state and initial control values are specified for each flight state. Default control matrices are defined based on the configuration: helicopter, tandem, coaxial, or tiltrotor.

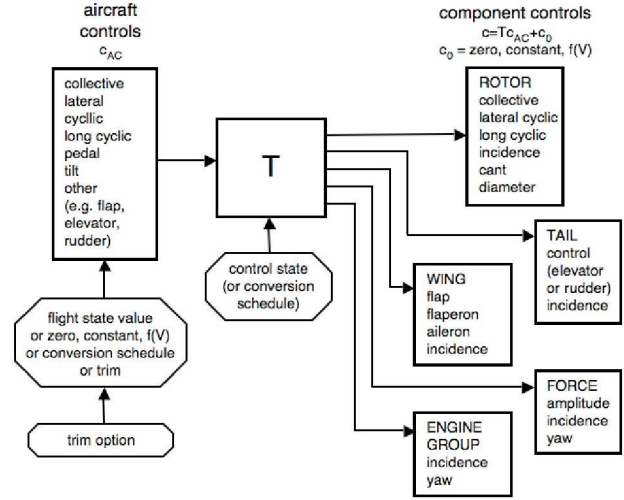


Figure 4. Aircraft and component controls

Typical (default) aircraft controls are the pilot's controls: collective stick, lateral and longitudinal cyclic sticks, pedal, and tilt. Units and sign convention of the pilot's controls are contained in the matrix  $T$ .

These aircraft controls are available for trim of the aircraft. Any aircraft controls not selected for trim will remain fixed at the values specified for the flight state. Thus by defining additional aircraft controls, component controls can be specified as required for a flight state.

The tilt control variable  $\alpha_{tilt}$  is intended for nacelle tilt angle or conversion control, particularly for tiltrotors. Typically this variable will be connected to nacelle/pylon, engine, rotor shaft, and even wings. The convention is  $\alpha_{tilt} = 0$  for cruise, and  $\alpha_{tilt} = 90$  deg for helicopter mode.

### Trim

The aircraft trim operation solves for the controls and motion that produce equilibrium in the specified flight state. In steady flight (including hover, level flight, climb and descent, and turns), equilibrium implies zero net force and moment on the aircraft. There can be additional quantities that at equilibrium must equal target values. In practice, the trim solution can deal with a subset of these quantities. Usually it is at least necessary to achieve equilibrium in the aircraft lift and drag forces, as well as in yaw moment for torque balance. The basic purpose of the trim solution is to determine the component states, including aircraft drag and rotor thrust, sufficient to evaluate the aircraft performance.

Different trim solution definitions are required for various flight states. Therefore one or more trim states are defined



for the analysis, and the appropriate trim state selected for each flight state of a performance condition or mission segment. For each trim state, the trim quantities, trim variables, and targets are specified. The available trim quantities include aircraft total force and moment, load factor, power or power margin, rotor force or torque, rotor flapping, wing force or lift margin, and tail force. The available trim variables include aircraft controls, orientation, speed, pullup rate, and turn rate.

### Geometry

Layout of the geometry is typically in terms of station line (SL, positive aft), buttline (BL, positive right), and waterline (WL, positive up), measured relative to some arbitrary origin. The  $x$ ,  $y$ , and  $z$  axes are parallel to the SL, BL, and WL directions. One or more locations are defined for each component of the aircraft. Each component will at least have a location that is the point where component forces and moments act on the aircraft. Each location is input in fixed or scaled form. The fixed form input is SL/BL/WL (dimensional). The scaled form input is  $x/L$  (positive aft),  $y/L$  (positive right), and  $z/L$  (positive up), from a reference point. The reference length  $L$  is the rotor radius or wing span of a designated component, or the fuselage length. Fixed input can be used for the entire aircraft, or just for certain components.

### Aircraft Motion

The aircraft velocity and orientation are defined by the flight speed, turn rate, orientation of the body frame relative to inertial axes (Euler angles), and orientation of the velocity frame relative to inertial axes (flight path angles).

Aircraft conventions are followed for the direction and orientation of axes: the  $z$ -axis is down, the  $x$ -axis forward, and the  $y$ -axis to the right; and a yaw-pitch-roll sequence is used for the Euler angles.

The orientation of the body frame  $F$  relative to inertial axes is defined by yaw  $\psi_F$ , pitch  $\theta_F$ , and roll  $\phi_F$  Euler angles. The flight path is specified by the velocity  $V$ , in the positive  $x$ -axis direction of the velocity axes. The orientation of the velocity axes relative to inertial axes is defined by yaw  $\psi_V$  (sideslip) and pitch  $\theta_V$  (climb) angles. In straight flight, all these angles are constant. In turning flight at a constant yaw rate, the yaw angle is  $\psi_F = \dot{\psi}_F t$ . The aircraft angular velocity  $\omega_{AC}^F$  is obtained from the Euler angle rates. For steady state flight,  $\dot{\theta}_F = \dot{\phi}_F = 0$ ;  $\dot{\psi}_F$  is nonzero in a turn. Accelerated flight is also considered, in terms of linear acceleration and pitch rate.

### Loads and Performance

For each component, the power required and the net forces and moments acting on the aircraft can be calculated. The aerodynamic forces  $F$  and moments  $M$  are typically calculated in wind axes and then resolved into body axes, relative to the origin of the body axes (the aircraft center-of-gravity). The power and loads of all components are summed to obtain the aircraft power and loads. Typically the trim solution drives the net forces and moments on the aircraft to zero.

The aircraft equations of motion, in body axes  $F$  with origin at the aircraft center-of-gravity, are the equations of force and moment equilibrium. The forces and moments are the sum of loads from all components of the aircraft: fuselage, rotor, force, wing, tail, engine, and tank. A particular component can have more than one source of loads; for example, the rotor component produces hub forces and moments, but also includes hub and pylon drag.

The component power required  $P_{comp}$  is evaluated for all components (rotors) of the propulsion group. The total power required for the propulsion group  $P_{reqPG}$  is obtained by adding the transmission losses and accessory power. The power required for the propulsion group must be distributed to the engine groups. From the power required the fuel flow is calculated. The fuel flow of the propulsion group is obtained from the sum over the engine groups. The total fuel flow is the sum from all components of the aircraft:  $\dot{W} = \sum \dot{W}_{reqEG} + \sum \dot{W}_{force}$

### Aerodynamics

Each component has a position  $z^F$  in aircraft axes  $F$ , relative to the reference point, and orientation of component axes  $B$  relative to aircraft axes given by a rotation matrix. Acting at the component are interference velocities  $v_{int}$  (velocity of air, in  $F$  axes) from all other components. Then the total component velocity relative to the air is the sum of velocity, angular velocity, and interference terms. The aerodynamic environment is defined in the component axes: velocity magnitude  $v = |v^B|$ , dynamic pressure  $q = \frac{1}{2} \rho v^2$ , angle-of-attack  $\alpha$ , and sideslip angle  $\beta$ . From these the component aerodynamic model calculates the loads in wind axes: drag, side force, and lift ( $D$ ,  $Y$ , and  $L$ ) and the roll, pitch, and yaw moments; and from wind axis loads the aerodynamic loads in aircraft axes acting at the center-of-gravity are calculated.

The component aerodynamic models are based on equations intended to cover the principal operating

conditions, including small angle-of-attack and sideslip with a representation of stall and maximum lift, vertical flight, sideward flight, and rearward flight.

### Drag

Each component can contribute drag to the aircraft. A fixed drag can be specified, as a drag area  $D/q$ ; or the drag can be scaled, specified as a drag coefficient  $C_D$  based on an appropriate area  $S$ . There may also be other ways to define a scaled drag value. For fixed drag, the coefficient is  $C_D = (D/q)/S$  (the aerodynamic model is formulated in terms of drag coefficient). For scaled drag, the drag area is  $D/q = SC_D$ . For all components, the drag  $(D/q)_{\text{comp}}$  or  $C_{D\text{comp}}$  is defined for forward flight or cruise; typically this is the minimum drag value. For some components, the vertical drag or sideward drag is defined. For some components, the aerodynamic model includes drag due to lift, angle-of-attack, or stall.

Table 1 summarizes the component contributions to drag, and the corresponding reference areas. If no reference area is indicated, then the input is only drag area  $D/q$ . An appropriate drag reference area is defined for each component; the reference area is either input or calculated.

Table 1. Component contributions to drag.

component	drag contribution	reference area
fuselage	fuselage	fuselage wetted area
	fuselage vertical	fuselage projected area
	fixtures and fittings	fuselage wetted area
	rotor-body interf	fuselage wetted area
	contingency	—
	payload increment	—
gear	landing gear	—
rotor	hub, hub vertical	rotor disk area
	pylon, pylon vert	pylon wetted area
	spinner	spinner wetted area
wing	wing, wing vert	wing planform area
	wing-body interf	wing planform area
tail	tail, tail vertical	tail planform area
engine	nacelle, nac. vert	nacelle wetted area
	momentum drag	—
fuel tank	auxiliary tank	—

Optionally the aircraft drag can be fixed. The quantity specified is the sum (over all components) of the drag area

$D/q$  (minimum drag, excluding drag due to lift and angle-of-attack), without accounting for interference effects on dynamic pressure. The input parameter can be  $D/q$ ; or the drag can be scaled, specified as a drag coefficient based on the rotor disk area, so  $D/q = A_{\text{ref}} C_D$  ( $A_{\text{ref}}$  is the reference rotor disk area); or the drag can be estimated from the aircraft maximum take-off weight,  $D/q = k(W_{\text{MTO}}/1000)^{2/3}$ . Based on historical data, the drag coefficient  $C_D = 0.02$  for old helicopters and  $C_D = 0.008$  for current low drag helicopters. Based on historical data,  $k = 9$  for old helicopters,  $k = 2.5$  for current low drag helicopters,  $k = 1.6$  for current tiltrotors, and  $k = 1.4$  for turboprop aircraft ( $k$  in English units). If the aircraft drag is input, then the fuselage contingency drag is adjusted so the total aircraft  $D/q$  equals the input value.

The nominal drag areas of the components and the aircraft are part of the aircraft description and are used when the aircraft drag is fixed. While vertical drag parameters are part of the aerodynamic model for the hub, pylon, and nacelle, aerodynamic interference at the rotor and at the engine group is not considered, so these terms do not contribute to download. In the context of download, only the fuselage, wing, tail, and contingency contribute to the nominal vertical drag.

### FUSELAGE

There is one fuselage component for the aircraft. There are several options for calculating the fuselage length, wetted area (reference area for drag coefficients), and projected area (reference area for vertical drag).

The aerodynamic velocity of the fuselage relative to the air, including interference, is calculated in component axes. The drag area or drag coefficient is defined for forward flight, vertical flight, and sideward flight. For small angle-of-attack, the drag increases proportional to  $(1 + K |\alpha|^X)$ , using input factor  $K$  and exponent  $X$ . In addition, the forward flight drag area or drag coefficient is defined for fixtures and fittings, and for rotor-body interference. The drag force is

$$D = qS_{\text{wet}}(C_D + C_{D\text{fit}} + \Sigma C_{D\text{rb}}) + q((D/q)_{\text{pay}} + (D/q)_{\text{cont}})$$

including the drag area of the payload (specified for the flight state) and the contingency drag area.

The fuselage lift, pitch moment, side force, and yaw moment are defined in fixed form (e.g.  $L/q$ ) or scaled form (e.g.  $C_L$ ). Maximum lift coefficient and maximum side force coefficient are defined.

## ROTOR

The aircraft can have one or more rotors, or no rotors. In addition to main rotors, the rotor component can model tail rotors, propellers, proprotors, ducted fans, thrust vectoring rotors, and auxiliary-thrust rotors. The principal configuration designation (main rotor, tail rotor, or propeller) determines where the weights are put in the weight statement (rotor group, empennage group, or propulsion group), and each configuration can possibly have a separate performance or weight model. Antitorque rotors and auxiliary-thrust rotors can be identified, for special sizing options. Other configuration features are variable diameter and ducted fan.

Multi-rotor systems (such as coaxial or tandem configuration) are modeled as a set of separate rotors, in order to accommodate the description of the position, orientation, controls, and loads. The performance calculation for twin rotor systems can include the mutual influence of the induced velocity on the power.

### Drive System

The drive system defines gear ratios for all the components that it connects (rotors and engine groups). There is one primary rotor per propulsion group (for which the reference tip speed is specified); other components are dependent (for which a gear ratio is specified). For the primary rotor, a reference tip speed  $V_{\text{tip-ref}}$  is defined for each drive system state.

### Geometry

The rotor rotation direction is described by the parameter  $r$ :  $r=1$  for counter-clockwise rotation,  $r=-1$  for clockwise rotation (as viewed from the positive thrust side of the rotor). The rotor solidity and blade mean chord are related by  $\sigma = Nc/\pi R$ . Generally thrust-weighted values are used, but geometric values are also required by the analysis. A general blade chord distribution is specified as  $c(r) = c_{\text{ref}}\hat{c}(r)$ , where  $c_{\text{ref}}$  is the thrust-weighted chord. Linear taper is specified in terms of a taper ratio  $t = c_{\text{tip}}/c_{\text{root}}$ , or in terms of the ratio of thrust-weighted and geometric chords,  $f = \sigma_t/\sigma_g = c_{.75R}/c_{.50R}$ .

The rotor hub is at position  $z_{\text{hub}}^F$ . A component of the position can be calculated, superseding the location input. The calculated geometry depends on the configuration. For a coaxial rotor, the hub locations can be calculated from the input separation, and the input location midway between the hubs. For a tandem rotor, the hub locations can be calculated from the input overlap, and the input location midway between the hubs. For a tail rotor, the

longitudinal position can be calculated from the main rotor radius, tail rotor radius, and tail-rotor/main-rotor clearance. For a tiltrotor, the lateral position can be calculated from the rotor radius (cruise value for variable diameter rotor), fuselage/rotor clearance, and fuselage width (with the pivot, pylon, and nacelle center-of-gravity lateral positions adjusted to keep the same relative position to the hub). Alternatively for a tiltrotor, the lateral position can be calculated from the wing span so the rotors are at the wing tips; or from a designated wing panel edge.

### Control and Loads

The rotor controls consist of collective, lateral cyclic, longitudinal cyclic, and perhaps shaft incidence (tilt) and cant angles. Rotor cyclic control can be defined in terms of tip-path plane or no-feathering plane command. The collective control variable is the rotor thrust amplitude  $T$  or  $C_T/\sigma$  (in shaft axes), from which the collective pitch angle can be calculated. This approach eliminates an iteration between thrust and inflow.

The relationship between tip-path plane tilt and hub moment is  $M = (N/2)I_b\Omega^2(\nu^2 - 1)\beta = K_{\text{hub}}\beta$ , where  $N$  is the number of blades,  $\Omega$  the rotor speed, and  $\nu$  the dimensionless fundamental flap frequency. The flap moment of inertia  $I_b$  is obtained from the input Lock number.

Optionally the rotor can have a variable diameter. The rotor diameter is treated as a component control. As a control it can be connected to an aircraft control and thus set for each flight state.

Tip-path plane command is characterized by direct control of the rotor thrust magnitude and the tip-path plane tilt. Cyclic tilt of the tip-path plane, hence tilt of the thrust vector, consists of longitudinal tilt  $\beta_c$  (positive forward) and lateral tilt  $\beta_s$  (positive toward retreating side). Alternatively, the cyclic control can be specified in terms of hub moment or lift offset, if the blade flap frequency is greater than 1/rev. This control mode requires calculation of rotor collective and cyclic pitch angles from the thrust magnitude and flapping.

No-feathering plane command is characterized by control of rotor cyclic pitch angles, and direct control of the rotor thrust magnitude. Cyclic tilt of the no-feathering plane, usually producing tilt of the thrust vector, consists of longitudinal cyclic pitch angle  $\theta_s$  (positive aft) and lateral cyclic pitch angle  $\theta_c$  (positive toward retreating side). This control mode requires calculation of rotor collective pitch angle and tip-path plane tilt from the thrust

magnitude and cyclic control, including the influence of inflow.

### Rotor Axes and Shaft Tilt

The rotor hub is at position  $z_{\text{hub}}^F$ , where the rotor forces and moments act; the orientation of the rotor shaft axes relative to the aircraft axes is given by the rotation matrix  $C^{SF}$ . The pivot is at position  $z_{\text{pivot}}^F$ . The hub or shaft axes  $S$  have origin at the hub center; the  $z$ -axis is the shaft, positive in the positive thrust direction; and the  $x$ -axis usually downstream. The rotor orientation is specified by selecting a nominal direction in body axes (positive or negative  $x$ ,  $y$ , or  $z$ -axis) for the positive thrust direction; the other two axes are then the axes of shaft control. For a main rotor the nominal direction would be the negative  $z$ -axis; for a tail rotor the nominal direction would be the  $\pm y$ -axis (depending on the direction of rotation of the main rotor); for a propeller the nominal direction would be the positive  $x$ -axis. The hub and pivot axes have a fixed orientation relative to the body axes: hub incidence and cant, pivot dihedral, pitch, and sweep. The shaft angle control consists of incidence and cant about the pivot axes.

For a tiltrotor aircraft, one of the aircraft controls is the nacelle angle, with the convention  $\alpha_{\text{tilt}} = 0$  for cruise, and  $\alpha_{\text{tilt}} = 90$  deg for helicopter mode. The rotor shaft incidence angle is then connected to  $\alpha_{\text{tilt}}$  by defining the control matrix appropriately.

### Hub Loads

The rotor controls give the thrust magnitude and the tip-path plane tilt angles  $\beta_c$  and  $\beta_s$ , either directly or from the collective and cyclic pitch. The forces acting on the hub are the thrust  $T$ , drag  $H$ , and side force  $Y$  (positive in  $z$ ,  $x$ , and  $y$ -axis directions respectively). The hub pitch and roll moment are proportional to the flap angles. The hub torque is obtained from the shaft power  $P_{\text{shaft}}$  and rotor speed  $\Omega$ . The force and moment acting on the hub, in shaft axes, are then:

$$F^S = \begin{pmatrix} H \\ Y \\ T \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ -f_B T \end{pmatrix}$$

$$M^S = \begin{pmatrix} M_x \\ M_y \\ -rQ \end{pmatrix} = \begin{pmatrix} K_{\text{hub}}(r\beta_s) \\ K_{\text{hub}}(-\beta_c) \\ -rP_{\text{shaft}}/\Omega \end{pmatrix}$$

The force includes a term equal to the rotor thrust times an input blockage factor  $f_B = \Delta T / T$ . This term accounts for blockage or download, as an alternative to including the

drag of the fuselage or a lifting surface in the aircraft trim. For example,  $f_B$  can model the tail rotor blockage caused by operation near the vertical tail.

The rotor loads in aircraft axes acting at the center-of-gravity ( $F^F$  and  $M^F$ ) are then calculated from the shaft axis loads ( $F^S$  and  $M^S$ ) and  $\Delta z = z_{\text{hub}}^F - z_{\text{cg}}^F$ .

The wind axes lift  $L$  and drag  $X$  are calculated from the net rotor hub force  $F^F$  and the rotor velocity  $v^F$ . The velocity relative to the air gives the propulsive force direction  $e_p = v^F / |v^F|$  (no interference), and the velocity magnitude  $V = |v^F|$ . The lift and drag components of the force are  $XV = -(v^F)^T F^F$  and  $L^2 = |F^F|^2 - X^2$ .

### Aerodynamics

The rotor velocity relative to the air is  $v^F = v_{AC}^F + \omega_{AC}^F \times \Delta z^F$  in aircraft axes. The velocities in shaft axes are

$$v^S = C^{SF} v^F = \Omega R \begin{pmatrix} -\mu_x \\ r\mu_y \\ \mu_z \end{pmatrix}$$

where  $\Omega R$  is the rotor tip speed. The advance ratio, inflow ratio, and shaft angle-of-attack are defined as  $\mu = (\mu_x^2 + \mu_y^2)^{1/2}$ ,  $\lambda = \lambda_i + \mu_z$ , and  $\alpha = \tan^{-1}(\mu_z / \mu)$ . The rotor thrust coefficient is defined as  $C_T = T / \rho A (\Omega R)^2$ . The dimensionless ideal induced velocity  $\lambda_i$  is calculated from  $\mu$ ,  $\mu_z$ , and  $C_T$ ; the dimensional velocity is  $v_i = \Omega R \lambda_i$ . The ideal induced power is then  $P_{\text{ideal}} = T v_i$ . Note that for these inflow velocities, the subscript “ $i$ ” denotes “ideal.”

### Ideal Inflow

The ideal wake-induced velocity is obtained from the momentum theory result of Glauert:

$$\lambda_i = \frac{C_T}{2\sqrt{\lambda^2 + \mu^2}} = \frac{s\lambda_h^2}{\sqrt{\lambda^2 + \mu^2}}$$

where  $\lambda = \lambda_i + \mu_z$ ,  $\lambda_h^2 = |C_T|/2$  ( $\lambda_h$  is always positive), and  $s = \text{sign}(C_T)$ . This expression is generalized to  $\lambda_i = \lambda_h s F(\mu / \lambda_h, s\mu_z / \lambda_h)$ . If  $\mu$  is zero, the equation for  $\lambda_i$  can be solved analytically. Otherwise, for non-axial flow, an iterative Newton-Raphson solution for  $\lambda_i$  is needed.

An approximate equation is used in the turbulent-wake and vortex-ring states to eliminate the singularity of the momentum theory result at ideal autorotation. The



momentum theory result is also extended to the case of a ducted fan.

The wake-induced velocity is reduced when the rotor disk is in the proximity of the ground plane,  $(\lambda_i)_{IGE} = f_g(\lambda_i)_{OGE}$ . The factor  $f_g$  is a function of the scaled rotor height above the ground,  $z_g/D$ . The effects of ground plane tilt and rotor velocity are modeled. Several empirical ground effect models are implemented.

As a simple approximation to nonuniform induced velocity distribution, a linear variation over the disk is used:  $\Delta\lambda = \lambda_x r \cos\psi + \lambda_y r \sin\psi$ . There are contributions to  $\Delta\lambda$  from forward flight and from hub moments, which influence the relationship between flapping and cyclic. The models implemented for the forward flight gradients are based on references 12 to 15. Differential momentum theory is used to calculate the gradients caused by hub moments.

#### Rotor Forces

Direct control of the rotor thrust magnitude is used, so the rotor collective pitch angle  $\theta_0$  must be calculated from the thrust  $C_T/\sigma$ . If the commanded variable were the collective pitch angle, then calculating the rotor thrust would be necessary, resulting in a more complicated solution procedure; in particular, an iteration between thrust and inflow would be needed. There may be flight states where the commanded thrust can not be produced by the rotor, even with stall neglected in the section aerodynamics. This situation will manifest as an inability to solve for the collective pitch given the thrust. In this circumstance the trim method should be changed so the required or specified thrust is an achievable value.

The inplane hub forces are produced by tilt of the thrust vector with the tip-path plane, plus forces in the tip-path plane, and profile terms (produced by the blade drag coefficient). The orientation of the tip-path axes relative to the shaft axes is  $C^{PS}$ , which is determined by  $\beta_c$  and  $\beta_s$ . Then

$$\begin{pmatrix} C_H \\ C_Y \\ C_T \end{pmatrix} = C^{SP} \begin{pmatrix} 0 \\ 0 \\ C_T/C_{33}^{SP} \end{pmatrix} + \begin{pmatrix} C_{Htp} \\ rC_{Ytp} \\ 0 \end{pmatrix} + \begin{pmatrix} C_{Ho} \\ rC_{Yo} \\ 0 \end{pmatrix}$$

The inplane forces relative to the tip-path plane can be neglected or calculated by blade element theory. Note that with tip-path plane command and  $C_{Htp}$  and  $C_{Ytp}$  neglected, solving for the rotor collective and cyclic pitch angles is not necessary. In general the inplane forces relative to the tip-path plane are not zero, and may be

significant, as for a rotor with large flap stiffness. Figure 5 shows the tip-path plane tilt and thrust vector tilt with cyclic pitch control (no-feathering plane tilt) as a function of flap stiffness (per-rev frequency) for several rotor thrust values. The difference between tip-path plane tilt and thrust vector tilt is caused by tilt of the thrust vector relative to the tip-path plane.

The profile inplane forces can be obtained from simplified equations, or calculated by blade element theory. The simplified method uses:

$$\begin{pmatrix} C_{Ho} \\ C_{Yo} \end{pmatrix} = \frac{\sigma}{8} c_{d\text{mean}} F_H \begin{pmatrix} \mu_x/\mu \\ -\mu_y/\mu \end{pmatrix}$$

where the mean drag coefficient  $c_{d\text{mean}}$  is from the profile power calculation. The function  $F_H$  accounts for the increase of the blade section velocity with rotor edgewise and axial speed.

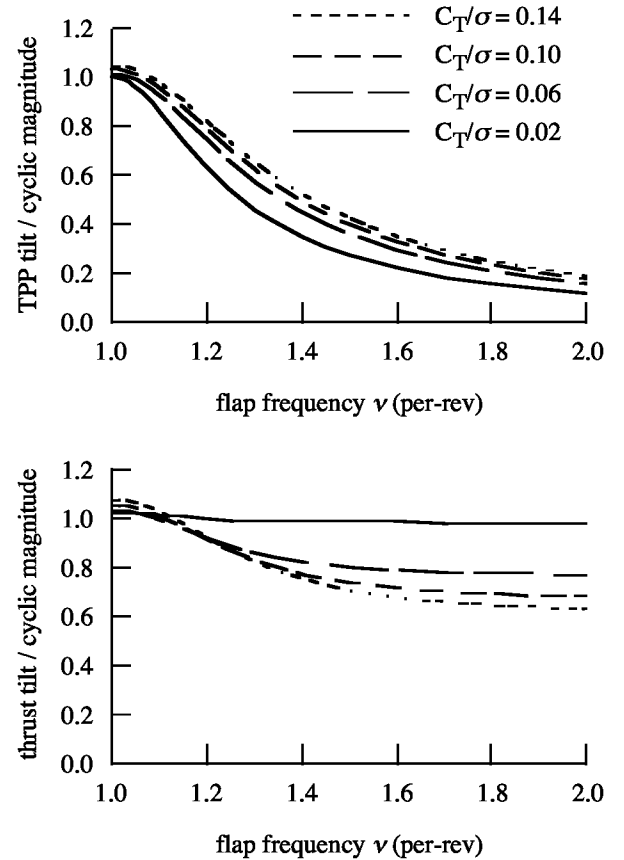


Figure 5. Tip-path plane tilt and thrust vector tilt with cyclic pitch.

### Blade Element Theory

Blade element theory is the basis for the solution for the collective and cyclic pitch angles (or flap angles) and evaluation of the rotor inplane hub forces. The section aerodynamics are described by lift varying linearly with angle-of-attack,  $c_l = c_{l\alpha}\alpha$  (no stall), and a constant mean drag coefficient  $c_{d\text{mean}}$  (from the profile power calculation). The blade section aerodynamic environment is described by the three components of velocity, from which the yaw and inflow angles are obtained, and then the angle-of-attack  $\alpha$ . The blade pitch consists of collective, cyclic, twist, and pitch-flap coupling terms. The flap motion is rigid rotation about a hinge with no offset, and only coning and once-per-revolution terms are considered. The inflow includes gradients caused by edgewise flight and hub moments. The effect of the inflow produced by hub moments is to introduce a lift-deficiency function in the flap response.

Integrating the section lift, drag, and radial forces over the blade radial coordinate and azimuth gives the total rotor thrust, drag, and side forces. Integrating terms produced by the section drag coefficient gives the profile inplane forces,  $C_{Ho}$  and  $C_{Yo}$ ; using blade element theory to evaluate these accounts for the planform and root cutout, while using  $F_H$  implies a rectangular blade and no root cutout (plus at most a 1% error approximating the exact integration). The remaining terms in the section forces produce the inplane loads relative to the tip-path plane,  $C_{H\text{tpp}}$  and  $C_{Y\text{tpp}}$ .

Evaluating these inplane forces requires the collective and cyclic pitch angles, and the flapping motion. The thrust equation must be solved for the rotor collective pitch. The relationship between cyclic pitch and flapping is defined by the rotor flap dynamics. The flap motion is rigid rotation about a central hinge, with a flap frequency  $\nu > 1$  for articulated or hingeless rotors. The flapping equation of motion, including the effects of precone and the inertial loads of shaft angular motion, is harmonically analyzed. The result is equations for the mean (coning) and 1/rev (tip-path plane tilt) flap motion. The solution for the coning is largely decoupled by introducing the rotor thrust coefficient.

The thrust and flapping equations of motion must be solved for the unknown angles. For tip-path plane command, the thrust and flapping are known, so the equations are solved for  $\theta_{0.75}$ ,  $\theta_c$ , and  $\theta_s$ . For no-feathering plane command, the thrust and cyclic pitch are known, so the equations are solved for  $\theta_{0.75}$ ,  $\beta_c$ , and  $\beta_s$ .

### Power

The rotor power consists of induced, profile, and parasite terms:  $P = P_i + P_o + P_p$ . The rotor parasite power (including climb/descent power for the aircraft) is obtained from the wind axis drag force:  $P_p = -XV = (v^F)^T F^F$ .

The induced power is calculated from the ideal power:  $P_i = \kappa P_{\text{ideal}} = \kappa f_D T v_{\text{ideal}}$ . The empirical factor  $\kappa$  accounts for the effects of nonuniform inflow, non-ideal span loading, tip losses, swirl, blockage, and other phenomenon that increase the induced power losses ( $\kappa > 1$ ). For a ducted fan,  $f_D = f_w / 2$  is introduced, where  $f_w$  is the ratio of the far wake induced velocity to the induced velocity at the disk.

The profile power is calculated from a mean blade drag coefficient:  $P_o = \rho A (\Omega R)^3 C_{Po}$ ,  $C_{Po} = (\sigma / 8) c_{d\text{mean}} F_P$ . The function  $F_P(\mu, \mu_z)$  accounts for the increase of the blade section velocity with rotor edgewise and axial speed.

Two performance methods are implemented: the energy method and the table method. The induced power factor and mean blade drag coefficient are obtained from equations with the energy method, or from tables with the table method. Optionally  $\kappa$  and  $c_{d\text{mean}}$  can be specified for each flight state, superseding the performance method values.

### Energy Performance Method: Induced Power

The induced power is calculated from the ideal power:  $P_i = \kappa P_{\text{ideal}} = \kappa f_D T v_{\text{ideal}}$ . Reference values of  $\kappa$  are specified for hover, axial cruise (propeller), and edgewise cruise (helicopter):  $\kappa_{\text{hover}}$ ,  $\kappa_{\text{prop}}$ ,  $\kappa_{\text{edge}}$ . Two models are implemented: constant model and standard model. The constant model uses  $\kappa = \kappa_{\text{hover}}$  if  $\mu = \mu_z = 0$ ; or  $\kappa = \kappa_{\text{prop}}$  if  $|\mu| < 0.1 |\mu_z|$ ; or  $\kappa = \kappa_{\text{edge}}$  otherwise.

The standard model calculates an axial flow factor  $\kappa_{\text{axial}}$  from the values  $\kappa_{\text{hover}}$ ,  $\kappa_{\text{climb}}$ , and  $\kappa_{\text{prop}}$ . Let  $\Delta = C_T / \sigma - (C_T / \sigma)_{\text{ind}}$ . For hover and low speed axial climb, including a variation with thrust, the inflow factor is

$$\kappa_h = \kappa_{\text{hover}} + k_{h1} \Delta_h + k_{h2} \Delta_h^2 + (\kappa_{\text{climb}} - \kappa_{\text{hover}}) \frac{2}{\pi} \tan^{-1} \left[ ((|\mu_z| / \lambda_h) / M_{\text{axial}})^{X_{\text{axial}}} \right]$$

where  $|\mu_z| / \lambda_h = M_{\text{axial}}$  is the midpoint of the transition between hover and climb. Figure 6 illustrates  $\kappa$  in hover (with a minimum value). A polynomial describes the variation with axial velocity, scaled so  $\kappa = \kappa_h$  at  $\mu_z = 0$

and  $\kappa = \kappa_p$  at  $\mu_z = \mu_{z\text{prop}}$ , including a variation with thrust:

$$\kappa_p = \kappa_{\text{prop}} + k_{p1}\Delta_p + k_{p2}\Delta_p^2$$

$$\kappa_{\text{axial}} = \kappa_h + k_{a1}\mu_z + S_a(k_{a2}\mu_z^2 + k_{a3}\mu_z^{X_a})$$

where  $S_a$  accomplishes the scaling. A polynomial describes the variation with edgewise advance ratio, scaled so  $\kappa = \kappa_{\text{axial}}$  at  $\mu = 0$  and  $\kappa = f_{\text{off}}\kappa_{\text{edge}}$  at  $\mu = \mu_{\text{edge}}$ . Thus the induced power factor is

$$\kappa = \kappa_{\text{axial}} + k_{e1}\mu + S_e(k_{e2}\mu^2 + k_{e3}\mu^{X_e})$$

where  $S_e$  accomplishes the scaling. The function  $f_{\text{off}} = 1 - k_{o1}(1 - e^{-k_{o2}o_x})$  accounts for the influence of lift offset,  $o_x = rM_x/TR = (K_{\text{hub}}/TR)\beta_s$ . Figure 7 illustrates  $\kappa$  in edgewise flight. Minimum and maximum values of the induced power factor are also specified.

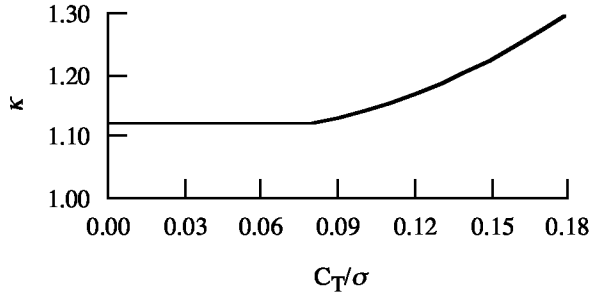


Figure 6. Induced power factor in hover (with minimum  $\kappa = 1.12$ ).

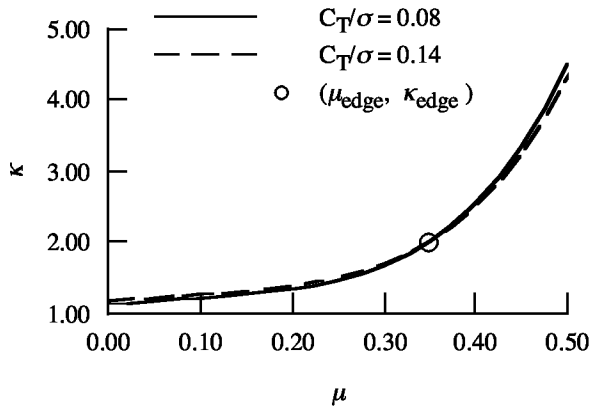


Figure 7. Induced power factor in edgewise flight.

#### Energy Performance Method: Profile Power

The profile power is calculated from a mean blade drag coefficient:  $C_{Po} = (\sigma/8)c_{d\text{mean}}F_P$ . Since the blade mean lift coefficient is  $c_l \approx 6C_T/\sigma$ , the drag coefficient is

estimated as a function of blade loading  $C_T/\sigma$  (using thrust-weighted solidity). With separate estimates of the basic, stall, and compressibility drag, the mean drag coefficient is

$$c_{d\text{mean}} = \chi S(c_{d\text{basic}} + c_{d\text{stall}} + c_{d\text{comp}})$$

where  $\chi$  is a technology factor. The factor  $S = (Re_{\text{ref}}/Re)^{0.2}$  accounts for Reynolds number effects on the drag coefficient;  $Re$  is based on the thrust-weighted chord,  $0.75V_{\text{tip}}$ , and the kinematic velocity of the flight state; and  $Re_{\text{ref}}$  corresponds to the input drag coefficient information. Array and equation models are implemented for the basic drag. In the array model the basic drag  $c_{d\text{basic}}$  is input as a function of  $C_T/\sigma$  (linearly interpolated array).

In the equation model the basic drag  $c_{d\text{basic}}$  is a quadratic function of  $C_T/\sigma$ , plus an additional term allowing faster growth at high (sub-stall) angles of attack. Let  $\Delta = |C_T/\sigma - (C_T/\sigma)_{D\text{min}}|$ , where  $(C_T/\sigma)_{D\text{min}}$  corresponds to the minimum drag; and  $\Delta_{\text{sep}} = |C_T/\sigma - (C_T/\sigma)_{\text{sep}}|$ . Values of the basic drag equation are specified for helicopter (hover and edgewise) and propeller (axial climb and cruise) operation:

$$c_{dh} = d_{0\text{hel}} + d_{1\text{hel}}\Delta + d_{2\text{hel}}\Delta^2 + d_{\text{sep}}\Delta_{\text{sep}}^{X_{\text{sep}}}$$

$$c_{dp} = d_{0\text{prop}} + d_{1\text{prop}}\Delta + d_{2\text{prop}}\Delta^2 + d_{\text{sep}}\Delta_{\text{sep}}^{X_{\text{sep}}}$$

The separation term is present only if  $\Delta_{\text{sep}} > 0$ . The helicopter and propeller values are interpolated as a function of  $\mu_z$ :

$$c_{d\text{basic}} = c_{dh} + (c_{dp} - c_{dh}) \frac{2}{\pi} \tan^{-1}(|\mu_z|/\lambda_h)$$

so  $|\mu_z|/\lambda_h = 1$  is the midpoint of the transition.

The stall drag increment represents the rise of profile power caused by the occurrence of significant stall on the rotor disk. Let  $\Delta_s = |C_T/\sigma - (f_s/f_{\text{off}})(C_T/\sigma)_s|$ , where  $f_s$  is an input factor. The function  $f_{\text{off}} = 1 - d_{o1}(1 - e^{-d_{o2}o_x})$  accounts for the influence of lift offset,  $o_x = rM_x/TR = (K_{\text{hub}}/TR)\beta_s$ . Then

$$c_{d\text{stall}} = d_{s1}\Delta_s^{X_{s1}} + d_{s2}\Delta_s^{X_{s2}}$$

(zero if  $\Delta_s \leq 0$ ). The blade loading at which the stall affects the entire rotor power,  $(C_T/\sigma)_s$ , is an input function of the velocity ratio  $V = (\mu^2 + \mu_z^2)^{1/2}$ .

Figure 8 shows typical stall functions  $(C_T/\sigma)_s$  for two rotors with different airfoils. For reference, typical rotor steady and transient load limits are also shown.

The compressibility drag increment depends on the advancing tip Mach number  $M_{at}$ . Drag divergence and

similarity models are implemented. For the drag divergence model, let  $\Delta M = M_{at} - M_{dd}$ , where  $M_{dd}$  is the drag divergence Mach number of the tip section. Then the compressibility increment in the mean drag coefficient is

$$c_{d \text{ comp}} = d_{m1} \Delta M + d_{m2} \Delta M^{X_m}$$

(ref. 16).  $M_{dd}$  is a function of the advancing tip lift coefficient (available from blade element theory) and the tip airfoil thickness-to-chord ratio.

Figure 9 illustrates the mean drag coefficient in hover, showing  $c_{dh}$  without and with the separation term, and the total for the high-stall and low-stall cases. Figure 10 illustrates the mean drag coefficient in forward flight, showing the compressibility term  $c_{d \text{ comp}}$ , and the growth in profile power with  $C_T/\sigma$  and  $\mu$  as the stall drag increment increases.

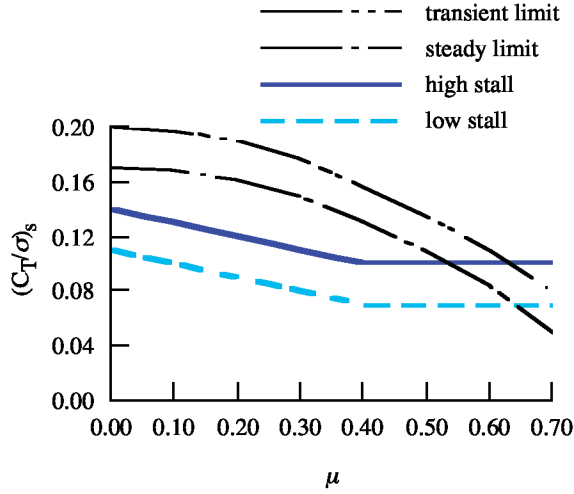


Figure 8. Profile power stall function.

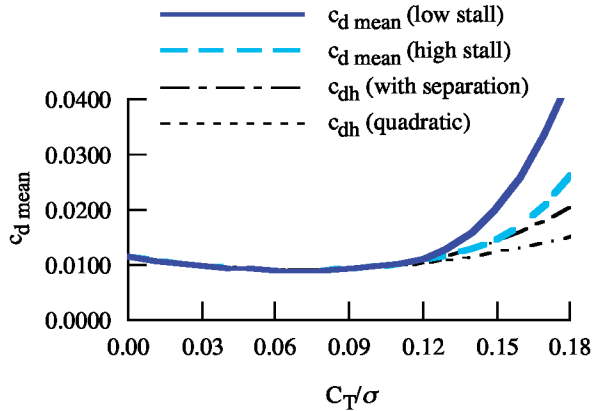


Figure 9. Mean drag coefficient in hover.

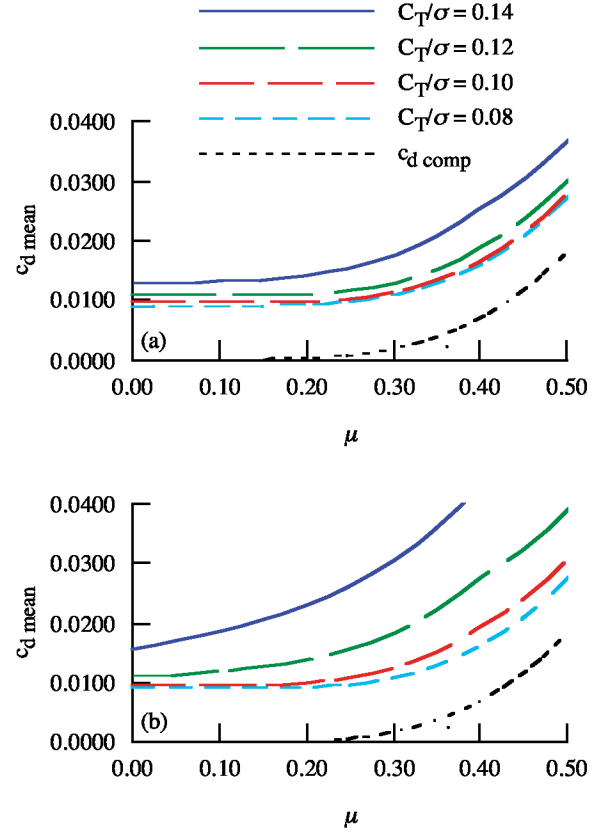


Figure 10. Mean drag coefficient in forward flight; (a) high stall; (b) low stall.

### Twin Rotors

For twin rotors, the induced power is determined by the induced velocity of the rotor system, not the individual rotors. The induced power is still obtained using  $P_i = \kappa P_{\text{ideal}} = \kappa f_D T v_{\text{ideal}}$  for each rotor, but the ideal induced velocity is calculated for an equivalent thrust  $C_{Te}$  based on the thrust and geometry of both rotors (see refs. 4 and 6). The profile power calculation is not changed for twin rotors.

### Interference

The rotor can produce aerodynamic interference velocities at the other components (fuselage, wings, tails). The induced velocity at the rotor disk is  $\kappa v_i$ , acting opposite the thrust ( $z$ -axis of tip-path plane axes). So  $v_{\text{ind}}^P = -k^P \kappa v_i$ , and  $v_{\text{ind}}^F = C^{FP} v_{\text{ind}}^P$ . The total velocity of the rotor disk relative to the air consists of the aircraft velocity and the induced velocity from this rotor:  $v_{\text{total}}^F = v^F - v_{\text{ind}}^F$ . The direction of the wake axis is thus  $e_w^P = -C^{PF} v_{\text{total}}^F / |v_{\text{total}}^F|$ . The angle of the wake axis from the thrust axis is  $\chi = \cos^{-1} |(k^P)^T e_w^P|$ .



The interference velocity  $v_{\text{int}}^F$  at each component is proportional to the induced velocity  $v_{\text{ind}}^F$  (and is in the same direction), with factors accounting for the stage of wake development and the position of the component relative to the rotor wake:

$$v_{\text{int}}^F = K_{\text{int}} f_W f_z f_r f_i v_{\text{ind}}^F$$

The factors  $f_W f_z$  account for axial development of the wake velocity, the factor  $f_r$  accounts for immersion in the wake, and  $K_{\text{int}}$  is an input empirical factor. An additional factor  $f_i$  for twin rotors is included.

Optionally the development along the wake axis can be a step function ( $f_W f_z = 0, 1$ , or  $f_W$  for above the rotor, on the rotor disk, or below the rotor disk, respectively); or the wake develops with the nominal or input rate of change. Optionally the wake immersion can use the contracted radius or the uncontracted radius; be a step function ( $f_r = 1$  and 0 inside and outside the wake boundary); be always immersed ( $f_r = 1$  always); or use an input transition distance.

The interference factor  $K_{\text{int}}$  can be reduced from an input value at low speed to zero at high speed, with linear variation over a specified speed range. To account for the extent of the wing or tail area immersed in the rotor wake, the interference velocity can be calculated at several points along the span and averaged.

### Drag

The rotor component includes drag forces acting on the hub and spinner and on the pylon.

The hub drag can be fixed, specified as a drag area  $D/q$ ; scaled, specified as a drag coefficient  $C_D$  based on the rotor disk area; or estimated based on the gross weight, using either a squared-cubed relationship or a square-root relationship. Based on historical data, the drag coefficient  $C_D = 0.004$  for typical hubs,  $C_D = 0.0024$  for current low drag hubs, and  $C_D = 0.0015$  for faired hubs. For the squared-cubed relationship:  $(D/q)_{\text{hub}} = k(W_{\text{MTO}}/1000)^{2/3}$  where  $W_{\text{MTO}}$  is the maximum take-off gross weight. Based on historical data,  $k = 1.4$  for typical hubs,  $k = 0.8$  for current low drag hubs, and  $k = 0.5$  for faired hubs ( $k$  in English units). For the square-root relationship:  $(D/q)_{\text{hub}} = k(W_{\text{MTO}}/N_{\text{rotor}})^{1/2}$  where  $W_{\text{MTO}}/N_{\text{rotor}}$  is the maximum take-off gross weight per lifting rotor. Based on historical data (ref. 17),  $k = 0.074$  for single rotor helicopters,  $k = 0.049$  for tandem rotor helicopters (probably a blade number effect),  $k = 0.038$  for hingeless rotors, and  $k = 0.027$  for faired hubs ( $k$  in English units).

The pylon forward flight drag and vertical drag are specified as drag area or drag coefficient, based on the pylon wetted area. The spinner drag is specified as drag area or drag coefficient, based on the spinner wetted area.

### FORCE

The force component is an object that can generate a force acting on the aircraft, possibly used for lift, or propulsion, or control. The amplitude of the force can be a fixed value, or connected to an aircraft control for trim. The direction of the force can be fixed, or connected to aircraft control.

The force generation requires a fuel flow that is calculated from an input thrust specific fuel consumption:  $\dot{w} = A(\text{sfc})$ , where  $A$  is the force amplitude, and the units of sfc are pound/hour/pound or kilogram/hour/Newton.

The force component weight is identified as either engine system or propeller/fan installation weight, both of the propulsion group. The force component weight is calculated from a specific weight and the design maximum force  $F_{\text{max}}$ , plus a fixed increment:  $W = SF_{\text{max}} + \Delta W$ .

### WING

The aircraft can have one or more wings, or no wings. The wing is described by planform area  $S$ , span  $b$ , mean chord  $c = S/b$ , and aspect ratio  $AR = b^2/S$ . These parameters are for the entire wing. The geometry is specified in terms of two of the following parameters:  $S$  or wing loading  $W/S$ ,  $b$  (perhaps calculated from other geometry),  $c$ ,  $AR$ . With more than one wing, the wing loading is obtained from an input fraction of design gross weight,  $W = f_W W_D$ . Optionally the span can be calculated from a specified ratio to the span of another wing.

For the tiltrotor configuration, the wing span can be calculated from the fuselage and rotor geometry (rotor radius, rotor-fuselage clearance, and fuselage width). The wing span can be calculated from the rotor hub position (regardless of how the rotor position is determined). Optionally the wing span can be calculated from an appropriate specification of all wing panel widths.

### Panels

The wing planform is defined in terms of one or more wing panels (figure 11). Symmetry of the wing is assumed. Each panel has a straight aerodynamic center and linear taper. The aerodynamic center locus (in wing axes) is defined by sweep, dihedral, and offsets at the inboard edge relative to the aerodynamic center of the previous panel. The wing position is the mean aerodynamic center.

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where the sum is over all other wings.

The wing interference at the tail produces an angle-of-attack change  $\varepsilon = E(C_L / C_{L\alpha})$ , where  $E = d\varepsilon / d\alpha$  is an input factor determined by the aircraft geometry. The change in orientation of the wing velocity produces the interference velocity  $v_{int}^F$  at the tail.

### Wing Extensions

The wing can have extensions, defined as wing portions of span  $b_x$  at each wing tip. For the tiltrotor configuration in particular, the wing weight depends on the distribution of wing area outboard (the extension) and inboard of the rotor and nacelle location. Wing extensions are defined as a set of wing panels at the tip. The extension span and area are the sum of the panel quantities.

### EMPENNAGE

The aircraft can have one or more tail surfaces, or no tail surface. Each tail is designated as horizontal or vertical. The tail is described by planform area  $S$ , span  $b$ , chord  $c = S/b$ , and aspect ratio  $AR = b^2/S$ . The tail volume  $V$  can be referenced to rotor radius and disk area; to wing area and chord for horizontal tails; or to wing area and span for vertical tails. The geometry is specified in terms of  $S$  or  $V$ , and  $b$  or  $AR$  or  $c$ .

The horizontal tail can have a cant angle  $\phi$  (positive tilt to left). The control variables are elevator and incidence.

The vertical tail can have a cant angle  $\phi$  (positive tilt to right). The control variables are rudder and incidence.

### FUEL TANK

The fuel tank capacity  $W_{fuel-cap}$  (maximum usable fuel weight) is determined from designated sizing missions. The maximum mission fuel required,  $W_{fuel-miss}$  (excluding reserves and any fuel in auxiliary tanks), gives

$$W_{fuel-cap} = \max(f_{fuel-cap} W_{fuel-miss}, W_{fuel-miss} + W_{reserve})$$

where  $f_{fuel-cap} \geq 1$  is an input factor. Alternatively, the fuel tank capacity  $W_{fuel-cap}$  can be input.

Auxiliary fuel tanks are defined in one or more sizes. The capacity of each auxiliary fuel tank,  $W_{aux-cap}$ , is an input parameter. The number of auxiliary fuel tanks on the aircraft,  $N_{auxtank}$  for each size, can be specified for the flight condition or mission segment. Alternatively (if the mission is not used to size the fuel tank), the number of auxiliary fuel tanks at the start of the mission can be determined from the mission fuel. The weight and drag of  $N_{auxtank}$  tanks are included in the performance calculation.

The weight of the auxiliary fuel tanks is an input fraction of the tank capacity. The drag area for each auxiliary tank is specified,  $(D/q)_{auxtank}$ .

### PROPULSION

The propulsion group is a set of components and engine groups connected by a drive system. The engine model describes a particular engine, which is used in one or more engine groups. The components (rotors) define the power required. The engine groups define the power available. Figure 12 illustrates the power flow.

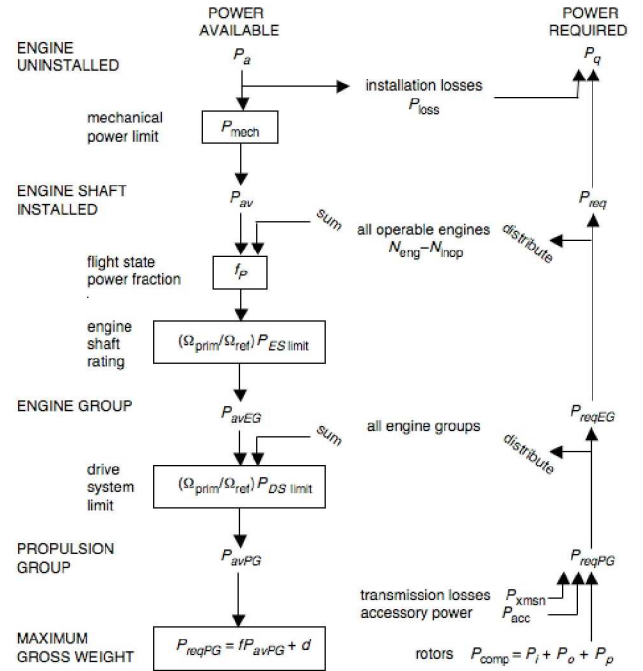


Figure 12. Power flow.

### Drive System

The drive system defines gear ratios for all the components that it connects. The gear ratio is the ratio of the component rotational speed to that of the primary rotor. There is one primary rotor per propulsion group (for which the reference tip speed is specified); other components are dependent (for which a gear ratio is specified). There can be more than one drive system state, in order to model a multiple-speed or variable-speed transmission. Each drive system state corresponds to a set of gear ratios.

For the primary rotor, a reference tip speed  $V_{tip-ref}$  is defined for each drive system state. By convention, the

“hover tip speed” refers to the reference tip speed for drive state #1. If the sizing task changes the hover tip speed, then the ratios of the reference tip speeds at different engine states are kept constant.

For dependent rotors, either the gear ratio is specified (for each drive system state), or a tip speed is specified and the gear ratio calculated ( $r = \Omega_{\text{dep}} / \Omega_{\text{prim}}$ ,  $\Omega = V_{\text{tip-ref}} / R$ ).

For the engine group, either the gear ratio is specified (for each drive system state), or the gear ratio calculated from the specification engine turbine speed  $\Omega_{\text{spec}} = (2\pi/60)N_{\text{spec}}$  and the reference tip speed of the primary rotor ( $r = \Omega_{\text{spec}} / \Omega_{\text{prim}}$ ,  $\Omega_{\text{prim}} = V_{\text{tip-ref}} / R$ ). The latter option means the specification engine turbine speed  $N_{\text{spec}}$  corresponds to  $V_{\text{tip-ref}}$  for all drive system states.

The flight state specifies the tip speed of the primary rotor and the drive system state, for each propulsion group. The drive system state defines the gear ratio for dependent rotors and the engine groups.

### Power Required

The component power required  $P_{\text{comp}}$  is evaluated for a specified flight condition, as the sum of the power required by all the components of the propulsion group. The total power required for the propulsion group is obtained by adding the transmission losses and accessory power:  $P_{\text{reqPG}} = P_{\text{comp}} + P_{\text{xmsn}} + P_{\text{acc}}$ . The power required for the propulsion group must be distributed to the engine groups. With only one engine group,  $P_{\text{reqEG}} = P_{\text{reqPG}}$ .

### ENGINE GROUP

The engine group consists of one or more engines of a specific type. For each engine type an engine model is defined.

The engine size is described by the power  $P_{\text{eng}}$ , which is the sea-level static power available per engine at a specified takeoff rating. The number of engines  $N_{\text{eng}}$  is specified for each engine group.

### Power Available

Given the flight condition and engine rating, the power available  $P_a$  is calculated (from the specific power  $SP_a$  and mass flow  $\dot{m}_a$ ). The flight condition information includes the altitude, temperature, flight speed, and primary rotor speed; a power fraction  $f_p$ ; and the states of the engine, drive system, and IR suppressor.

In the engine model, installation losses  $P_{\text{loss}}$  are subtracted from  $P_a$  ( $P_{\text{av}} = P_a - P_{\text{loss}}$ ), and then the mechanical limit applied:  $P_{\text{av}} = \min(P_{\text{av}}, P_{\text{mechR}})$ .

The engine model gives the performance of a single engine. The power available of the engine group is obtained by multiplying the single engine power by the number of engines operational (total number of engines less inoperable engines):  $P_{\text{avEG}} = (N_{\text{eng}} - N_{\text{inop}})P_{\text{av}}$ . The propulsion group power available is obtained from the sum over the engine groups:  $P_{\text{avPG}} = \sum f_p P_{\text{avEG}}$ , including a specified power fraction  $f_p$ .

The drive system rating at the flight condition is  $(\Omega_{\text{prim}} / \Omega_{\text{ref}})P_{\text{DSlimit}}$ . Optionally this limit is applied to the propulsion group power:  $P_{\text{avPG}} = \min(P_{\text{avPG}}, P_{\text{DSlimit}})$ . Similarly the engine shaft limit at the flight condition is optionally applied to the engine group power.

### Performance at Power Required

The engine performance (mass flow, fuel flow, and gross jet thrust) is calculated for a specified power required  $P_q$ , flight condition, and engine rating. The flight condition includes the altitude, temperature, flight speed, and primary rotor speed; and engine, drive system, and IR suppressor states. The engine turbine speed is  $N_{\text{eng}} = r_{\text{eng}}\Omega_{\text{prim}}$ .

The engine model deals with a single engine. The power required of a single engine is obtained by dividing the engine group power by the number of engines operational (total number of engines less inoperable engines):  $P_{\text{req}} = P_{\text{reqEG}} / (N_{\text{eng}} - N_{\text{inop}})$ . In the engine model, installation losses  $P_{\text{loss}}$  are added to  $P_{\text{req}}$  ( $P_q = P_{\text{req}} + P_{\text{loss}}$ ).

The performance of the engine group is obtained by multiplying the single engine characteristics by the number of engines operational (total number of engines less inoperable engines):

$$\begin{aligned}\dot{m}_{\text{reqEG}} &= (N_{\text{eng}} - N_{\text{inop}})\dot{m}_{\text{req}} \\ \dot{w}_{\text{reqEG}} &= (N_{\text{eng}} - N_{\text{inop}})\dot{w}_{\text{req}}K_{\text{ffd}} \\ F_{\text{N EG}} &= (N_{\text{eng}} - N_{\text{inop}})F_{\text{N}} \\ D_{\text{auxEG}} &= (N_{\text{eng}} - N_{\text{inop}})D_{\text{aux}}\end{aligned}$$

The fuel flow has also been multiplied by a factor  $K_{\text{ffd}}$  accounting for deterioration of the engine efficiency.

### Installation

The difference between installed and uninstalled power is the inlet and exhaust losses  $P_{\text{loss}}$ :  $P_{\text{av}} = P_a - P_{\text{loss}}$  and  $P_q = P_{\text{req}} + P_{\text{loss}}$ . The inlet ram recovery efficiency  $\eta_d$  is included in the engine model calculations. The inlet and exhaust losses are modeled as fractions of power available



or power required. Installation effects on the jet thrust are included in the engine model.

The momentum drag of the auxiliary air flow is a function of the mass flow  $\dot{m}_{aux} = f_{aux} \dot{m}_{req}$ :

$$D_{aux} = (1 - \eta_{aux}) \dot{m}_{aux} V = (1 - \eta_{aux}) f_{aux} \dot{m}_{req} V$$

where  $\eta_{aux}$  is the ram recovery efficiency.

### Control and Loads

The engine orientation is specified by selecting a nominal direction in body axes (usually thrust forward), then applying a yaw angle, then an incidence or tilt angle. The yaw and incidence angles can be connected to the aircraft controls. Hence the incidence and yaw angles can be fixed orientation or can be control variables.

The engine group produces a jet thrust  $F_N$ , acting in the direction of the engine; a momentum drag  $D_{aux}$ , acting in the wind direction; and a nacelle drag  $D_{nac}$ , acting in the wind direction.

### REFERRED PARAMETER TURBOSHAFT ENGINE MODEL (RPTEM)

Aircraft gas turbine engine performance capabilities are formally specified by computer programs known as engine decks, which are created by engine manufacturers in an industry-standard format. Engine decks are typically based on thermodynamic cycle analysis using real engine component performance maps. The most important performance maps for turboshaft engines are compressor, gas generator turbine, and power turbine. These component performance maps are critical to obtaining realistic off-design engine performance. Design and analysis codes calculate aircraft performance for a very wide range of operating conditions. This means that engine performance must be realistic even far from the engine design point. A simple thermodynamic cycle analysis that assumes design point component efficiencies everywhere is not realistic for such an application. Rather than developing models for component performance, a model for the total engine performance is used. The engine is not being designed.

The Referred Parameter Turboshaft Engine Model (RPTEM) is based on curve-fits of engine performance data for existing or projected engines, over a range of operating conditions. The curve-fits are typically obtained by exercising an engine deck (a computer program). The use of referred parameters tends to collapse the data, and provides a basis for scaling the engine. The operating condition is described by pressure altitude, ambient air

temperature, flight Mach number, power turbine speed, exhaust nozzle area, and either engine rating or engine power required. These curve-fits, typically based on real engines, are scaled to the required size and adjusted to the appropriate technology level to represent a notional engine. Engine size is represented by mass flow. Engine technology is represented by specific power available and specific fuel consumption at maximum continuous power (MCP), sea level/standard day (SLS), static (zero airspeed) conditions. Engine installation effects (inlet and exhaust losses) are also modeled.

The use of referred parameters to curve-fit engine performance data was suggested by David Woodley from Boeing during the JVX program (1983). The RPTEM was developed and documented by Michael P. Scully and Henry Lee of ASRAO, U.S. Army Aeroflightdynamics Directorate (AFDD), with a subsequent implementation written by Sam Ferguson (1995).

### Operating Environment

The operating condition and atmosphere give the standard conditions (temperature and pressure) for a specified pressure altitude; the sea-level standard conditions (temperature  $T_0$  and pressure  $p_0$ ); and the operating temperature  $T$  and pressure  $p = p_{std}$ . Here the temperatures are in deg R or deg K. The engine characteristics depend on the temperature ratio  $\theta = T/T_0$  and pressure ratio  $\delta = p/p_0$ . The flight Mach number  $M$  is obtained from the aircraft speed.

The inlet ram air temperature ratio and pressure ratio are obtained from  $M$  and the inlet ram recovery efficiency  $\eta_d$ :  $\theta_M = (1 + 0.2M^2)$  and  $\delta_M = (1 + \eta_d 0.2M^2)^{3.5}$ , using the ratio of specific heats  $\gamma = 1.4$ .

### Engine Ratings

The power available from a turboshaft engine depends on the engine rating. Each engine rating has specific operating limitations, most importantly an operating time limit intended to avoid damage to the engine. Typical engine ratings are MCP (maximum continuous power, no time limit), IRP (intermediate rated power, 30 min), MRP (maximum rated power, 10 min), and CRP (contingency rated power).

### Performance Characteristics

The engine performance is described by the power available  $P_a$ , at each engine rating and the specification engine turbine speed  $N_{spec}$ ; the mass flow  $\dot{m}$  and fuel flow  $\dot{w}$  required to produce power required  $P_q$  at engine

turbine speed  $N$ ; and the gross jet thrust  $F_g$  at a given power required  $P_q$ . Then the specific power is  $SP = P/\dot{m}$ , and the specific fuel consumption is  $sfc = \dot{w}/P$ .

The reference performance is at sea-level-standard static conditions (subscript 0), and MCP (subscript C). Referred or corrected engine parameters are used in the model: power  $P/(\delta\sqrt{\theta})$ , mass flow  $\dot{m}/(\delta/\sqrt{\theta})$ , specific power  $(P/\dot{m})/\theta$ , fuel flow  $\dot{w}/(\delta\sqrt{\theta})$ , thrust  $F/\delta$ , and turbine speed  $N/\sqrt{\theta}$ .

For each rating R, the performance is characterized by the following quantities for sea-level-standard static conditions: power  $P_{0R}$ , specific power  $SP_{0R}$ , and mechanical power limit  $P_{mechR}$ . The mass flow is then  $\dot{m}_{0R} = P_{0R}/SP_{0R}$ . The gross jet thrust  $F_{g0C}$  is given at MCP. These characteristics are at the specification turbine speed  $N_{spec}$ .

The installed power required  $P_{req}$  and power available  $P_{av} > P_{req}$  are measured at the engine output shaft. In addition to shaft power, the engine exhaust produces a net jet thrust  $F_N$ , from mass flow that goes through the engine core. The fuel flow and mass flow are the total required to produce the shaft power and jet thrust. The forces produced by mass flow that does not go through the engine core (such IR suppressor or cooling air) are treated as momentum drag  $D_{aux}$ . The relationship between net and gross jet thrust is  $F_n = F_g - \dot{m}_{req}V = \dot{m}_{req}(V_j - V)$ , where  $V_j$  is the engine jet exhaust velocity.

The uninstalled power required is  $P_q$ , the power available  $P_a$ , the gross jet thrust  $F_g$ , and net jet thrust  $F_n$ . The engine model calculates  $P_a$  as a function of flight condition and engine rating. Installation losses  $P_{loss}$  are subtracted from  $P_a$  ( $P_{av} = P_a - P_{loss}$ ), and then the mechanical limit applied:  $P_{av} = \min(P_{av}, P_{mechR})$ .

The engine performance (mass flow, fuel flow, and gross jet thrust) is calculated for a specified power required  $P_q$  (which might equal the power available), flight condition, and engine rating. Installation losses  $P_{loss}$  are added to  $P_{req}$  ( $P_q = P_{req} + P_{loss}$ ).

### Power Turbine Speed

The shaft power available is a function of the gas power available  $P_G$  and the power turbine efficiency  $\eta_t$ :  $P_a = \eta_t P_G$ . Generally the power turbine speed  $N$  has a significant effect on  $\eta_t$ , but almost no effect on  $P_G$ . The model used for the efficiency variation is

$$\eta_t \approx 1 - |(N/N_{opt}) - 1|^X$$

where  $N_{opt}$  is the speed for peak efficiency. Alternatively, the model parameters are defined at a set of engine speed ratios  $N/N_{spec}$ . Then the engine performance quantities at the required engine speed  $N$  are obtained by linear interpolation.

### Scaling

The parameters of the engine model can be defined for a specific engine, but as part of the aircraft sizing task the parameters must be scaled, in order to define an engine for a specified power. In addition, advanced technology must be represented in the model. Scaling and advanced technology are handled in terms of specific power and specific fuel consumption (at SLS static conditions, MCP, and  $N_{spec}$ ).

The engine size is specified as takeoff power  $P_{to} = P_{eng}$ : power at rating R, for SLS static conditions and specification turbine speed  $N_{spec}$ . Hence the MCP is  $P_{0C} = P_{to}/r_{p0R}$ , and the power at all other ratings follows.

The engine technology parameters  $SP_{0C}$  and  $sfc_{0C}$  are assumed to vary linearly with mass flow  $\dot{m}_{0C}$  up to a limit  $\dot{m}_{lim}$ , and constant thereafter at  $SP_{lim}$  and  $sfc_{lim}$ . Usually the effect of size is that specific power increases and specific fuel consumption decreases with mass flow. The specific thrust available at MCP is assumed to be constant, and the specification power turbine speed decreases with the square-root of the mass flow.

## WEIGHTS

The design gross weight  $W_D$  is a principal parameter defining the aircraft, usually determined by the sizing task for the design conditions and missions. The aircraft weight statement defines the empty weight, fixed useful load, and operating weight for the design configuration. The definition of the weight terms is as follows.

$$\text{operating weight: } W_O = W_E + W_{FUL}$$

$$\text{useful load: } W_{UL} = W_{FUL} + W_{pay} + W_{fuel}$$

$$\text{gross weight: } W_G = W_E + W_{UL} = W_O + W_{pay} + W_{fuel}$$

where  $W_E$  is the weight empty;  $W_{FUL}$  the fixed useful load;  $W_{pay}$  the payload weight; and  $W_{fuel}$  the usable fuel weight. The weight empty consists of structure, propulsion group, systems and equipment, vibration, and contingency weights. If the weight empty is input, then the contingency weight is adjusted so  $W_E$  equals the required value. If the design gross weight is input, then the payload or fuel weight must be fallout.

Table 2. Weight statement (\* indicates extension of RP8A).

<b>WEIGHT EMPTY</b>
<b>STRUCTURE</b>
wing group
rotor group
empennage group
fuselage group
alighting gear group
engine section or nacelle group
air induction group
<b>PROPULSION GROUP</b>
engine system
propeller/fan installation
fuel system
drive system
<b>SYSTEMS AND EQUIPMENT</b>
flight controls group
auxiliary power group
instruments group
hydraulic group
pneumatic group
electrical group
avionics group (mission equipment)
armament group
furnishings & equipment group
environmental control group
anti-icing group
load & handling group
<b>VIBRATION (*)</b>
<b>CONTINGENCY</b>
<b>FIXED USEFUL LOAD</b>
crew
fluids (oil, unusable fuel) (*)
auxiliary fuel tanks
other fixed useful load (*)
kits (*)
<b>PAYLOAD</b>
<b>USABLE FUEL</b>
standard tanks (*)
auxiliary tanks (*)

The gross weight  $W_G$  is specified for each flight condition and mission, perhaps in terms of the design gross weight  $W_D$ . For a each flight state, the fixed useful load may be different than the design configuration, because of changes in auxiliary fuel tank weight or kit weights or increments in crew or furnishings weights. Thus the fixed useful load weight is calculated for the flight state; and from it the

useful load weight and operating weight. The gross weight, payload weight, and usable fuel weight (in standard and auxiliary tanks) completes the weight information for the flight state.

### Weight Groups

Aircraft weight information is stored in a data structure that follows SAWE RP8A Group Weight Statement format (ref. 18), as outlined in table 2. The asterisks designate extensions of RP8A. There are 2 or 3 additional levels in the data structure for some weight groups, based on the weight breakdown for parametric estimation.

For each weight group, fixed (input) weights can be specified or weight increments added to the results of the parametric weight model. The parametric weight model includes technology factors. Weights of individual elements in a group can be fixed by setting the corresponding technology factor to zero.

The vibration control weight can be input, or specified as a fraction of weight empty:  $W_{vib} = f_{vib}W_E$ . The contingency weight can be input, or specified as a fraction of weight empty:  $W_{cont} = f_{cont}W_E$ . However, if the weight empty is input, then the contingency weight is adjusted so  $W_E$  equals the required value.

### AFDD Weight Models

For scaled weights of all components, the rotorcraft weight models developed by the U.S. Army Aeroflightdynamics Directorate (AFDD) are implemented. The weights are estimated from parametric equations based on the weights of existing turbine-powered helicopters and tiltrotors (and some fixed wing aircraft component weights). For some weight groups two models are available, designated AFDDnn. Table 3 summarizes the statistics of the parametric weight estimation equations.

Figure 13 shows the error of the calculated weight for the sum of all parametric weight, accounting on average for 42% of the empty weight. This sum is composed of the structural group (based on the AFDD00 equation for rotor blade and hub weights, and the AFDD84 equation for body weight), the propulsion group (based on the AFDD00 equation for drive system weight), and the flight controls group. Based on 42 aircraft, the average error of the sum of all parametric weight is 5.3%. The corresponding average error is 6.1% for the structural group (8.6% for the rotor group alone), 10.9% for the propulsion group, and 8.7% for the flight controls group.

Table 3. Statistics of parametric weight equations.

group		number of aircraft	average error %
wing		25	3.4
rotor blade	AFDD82	37	7.7
rotor hub	AFDD82	37	10.2
rotor blade	AFDD00	51	7.9
rotor hub	AFDD00	51	9.2
horizontal tail		13	22.4
vertical tail		12	23.3
tail rotor		19	16.7
fuselage	AFDD82	30	8.7
fuselage	AFDD84	35	6.5
alighting gear		28	8.4
engine support		12	11.0
engine cowling		12	17.9
air induction		12	11.0
accessory		16	11.5
fuel tank		15	4.6
gear box + rotor shaft	AFDD83	30	7.7
gear box + rotor shaft	AFDD00	52	8.6
drive shaft		28	16.0
rotor brake		23	25.1
rotary wing flight controls	non-boosted	20	10.4
rotary wing flight controls	boost mechanisms	21	6.5
rotary wing flight controls	boosted	20	9.7

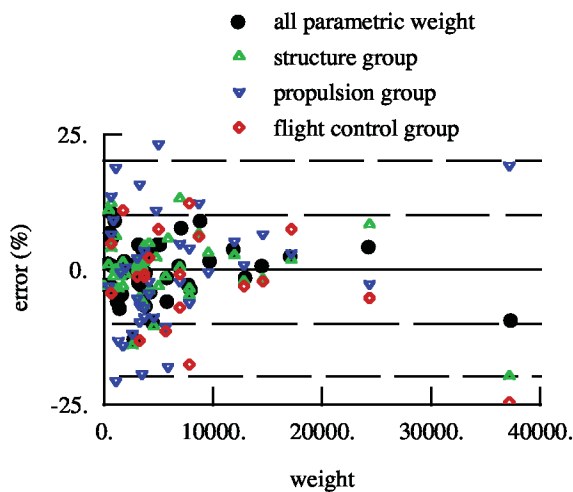


Figure 13. Accuracy of sum of all parametric weight.

## NDARC SOFTWARE

The NDARC program is entirely new software, built on an architecture that enables routine extensions and modifications. The software has been implemented with low-fidelity models, typical of the conceptual design environment. Incorporation of higher-fidelity models will be possible, as the architecture of the code accommodates configuration flexibility and a hierarchy of models.

The program is written in Fortran 95, using a special-purpose software tool to manage the data structures, construct the input manual, and automatically generate some input and output subroutines. The program has been compiled on several platforms and operating systems.

On typical computers, NDARC execution times range from seconds for a job with just a few analysis tasks, to minutes for a job that sizes an aircraft based on multiple flight conditions and missions.

Input is in namelist-based text format. The program output includes text files formatted for printing and for spreadsheets, and special files to support functions such as preparing layout drawings. Java graphical user interfaces are being developed by the user community to facilitate dealing with the input and output.

The program is supported by complete and thorough documentation, including the theory manual (Ref. 6), input manual, and data structures manual. A NASA-hosted Wiki has been established to support user communication.

Distribution of the NDARC program is controlled by the Software Release Authority at NASA Ames Research Center. Source code and documentation are available to users, subject to a Software Usage Agreement.

## CONCLUDING REMARKS

The theoretical basis and architecture of the conceptual design tool NDARC (NASA Design and Analysis of Rotorcraft) has been described. The principal tasks of NDARC are to design a rotorcraft to satisfy specified design conditions and missions, and then analyze the performance of the aircraft for a set of off-design missions and point operating conditions. NDARC provides a capability to model general rotorcraft configurations, and estimate the performance and attributes of advanced rotor concepts.



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## NOMENCLATURE

### Acronyms

AFDD	U.S. Army Aeroflightdynamics Directorate
IRP	intermediate rated power
ISA	International Standard Atmosphere
MCP	maximum continuous power
MRP	maximum rated power
RPTEM	referred parameter turboshaft engine model

### Weights

$W_D$	design gross weight
$W_E$	empty weight
$W_{MTO}$	maximum takeoff weight
$W_{SD}$	structural design gross weight
$W_G$	gross weight, $W_E + W_{UL} = W_O + W_{pay} + W_{fuel}$
$W_O$	operating weight, $W_E + W_{FUL}$
$W_{UL}$	useful load, $W_{FUL} + W_{pay} + W_{fuel}$
$W_{pay}$	payload
$W_{fuel}$	fuel weight
$W_{FUL}$	fixed useful load
$W_{burn}$	mission fuel burn
$W_{vib}$	vibration control weight
$W_{cont}$	contingency weight

### Fuel Tanks

$W_{fuel-cap}$	fuel capacity, maximum usable fuel weight
$N_{auxtank}$	number of auxiliary fuel tanks
$W_{aux-cap}$	auxiliary fuel tank capacity

### Power

$P_{reqPG}$	power required, propulsion group; $P_{comp} + P_{xmsn} + P_{acc}$
$P_{reqEG}$	power required, engine group
$P_{avPG}$	power available, propulsion group; $\min(\Sigma f_P P_{avEG}, (\Omega_{prim} / \Omega_{ref}) P_{DSLlimit})$
$P_{avEG}$	power available, engine group; $(N_{eng} - N_{inop}) P_{av}$

$P_{\text{comp}}$	component power required
$P_{\text{xmsn}}$	transmission losses
$P_{\text{acc}}$	accessory power
$N_{\text{inop}}$	number of inoperative engines, engine group
$P_{DS \text{ limit}}$	drive system torque limit (specified as power limit at reference rotor speed)
$P_{ES \text{ limit}}$	engine shaft rating

### Engine

$P_{\text{eng}}$	sea level static power available per engine at specified takeoff rating
$N_{\text{eng}}$	number of engines in engine group
$P_{\text{av}}$	power available, installed; $\min(P_a - P_{\text{loss}}, P_{\text{mech}R})$
$P_a$	power available, uninstalled
$P_{\text{req}}$	power required, installed; $P_q - P_{\text{loss}}$
$P_q$	power required, uninstalled
$P_{\text{loss}}$	installation losses
$P_{\text{mech}R}$	mechanical power limit
$SP$	specific power, $P/\dot{m}$ (conventional units)
sfc	specific fuel consumption, $\dot{w}/P$ (conventional units)
$\dot{m}$	mass flow (conventional units)
$\dot{w}$	fuel flow (conventional units)
$F_N$	net jet thrust
$D_{\text{aux}}$	momentum drag
$N$	specification turbine speed

### Tip Speed and Rotation

$V_{\text{tip-ref}}$	reference tip speed, propulsion group primary rotor; each drive state
$r$	gear ratio; $\Omega_{\text{dep}}/\Omega_{\text{prim}}$ for rotor, $\Omega_{\text{spec}}/\Omega_{\text{prim}}$ for engine
$\Omega_{\text{prim}}$	primary rotor rotational speed, $V_{\text{tip-ref}}/R$
$\Omega_{\text{dep}}$	dependent rotor rotational speed
$\Omega_{\text{spec}}$	specification engine turbine speed
$N_{\text{spec}}$	specification engine turbine speed (rpm)

### Axis Systems

I	inertial
F	aircraft

### Aerodynamics and Loads

$v$	velocity relative air (including interference)
$q$	dynamic pressure, $\frac{1}{2}\rho v^2$
$\alpha$	angle-of-attack
$\beta$	sideslip angle
$F$	force
$M$	moment
$D, Y, L$	drag, side, lift forces (aerodynamic axes)
$C_D, C_Y, C_L$	drag, side, lift force coefficients

$D/q$	drag area, $SC_D$ ( $S$ = reference area)
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### Aircraft

$DL$	disk loading, $W_D/A_{\text{ref}}$
$A_{\text{ref}}$	reference rotor area, $\Sigma f_A A$ ; typically projected area of lifting rotors
$WL$	wing loading, $W_D/S_{\text{ref}}$
$S_{\text{ref}}$	reference wing area, $\Sigma S$ ; sum area all wings
$c_{AC}$	aircraft control
$T$	control matrix
$c$	component control, $c = STc_{AC} + c_0$
$\alpha_{\text{tilt}}$	tilt control variable

### Rotor

$W/A$	disk loading, $W = f_W W_D$
$C_W/\sigma$	design blade loading, $W/\rho A V_{\text{tip}}^2 \sigma$ ( $V_{\text{tip}}$ = hover tip speed)
$R$	blade radius
$A$	disk area
$\sigma$	solidity (ratio blade area to disk area)
$T_{\text{design}}$	design thrust of antitorque or auxiliary-thrust rotor
$r$	direction of rotation ( $r = 1$ for counter-clockwise, $r = -1$ for clockwise)
$\mu$	advance ratio
$\lambda$	inflow ratio
$M_{\text{at}}$	advancing tip Mach number
$\nu$	blade flap frequency (per-rev)
$\gamma$	blade Lock number
$C_T/\sigma$	thrust coefficient divided by solidity, $T/\rho A (\Omega R)^2 \sigma$
$\beta_c, \beta_s$	longitudinal, lateral flapping (tip-path plane tilt relative shaft)
$\theta_{0.75}$	blade collective pitch angle (at 75% radius)
$\theta_c, \theta_s$	lateral, longitudinal blade pitch angle
$H, Y, T$	drag, side, thrust force on hub (shaft axes)
$M_x, M_y$	roll, pitch moment on hub
$Q$	shaft torque
$P_i, P_o, P_p$	induced, profile, parasite power
$\kappa$	induced power factor, $P_i = \kappa P_{\text{ideal}}$
$c_{d \text{ mean}}$	profile power mean drag coefficient, $C_{P_o} = (\sigma/8)c_{d \text{ mean}} F_P$

### Wing

$W/S$	wing loading, $W = f_W W_D$
$S$	area
$b$	span
$c$	chord, $S/b$
$AR$	aspect ratio, $b^2/S$